

## The Study of Hydraulic Properties of Quarter-Circular Crested Stepped Spillway

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### Abstract

In this paper, the quarter-circular crested stepped spillway (QCSS) is introduced and its hydraulic properties including the discharge coefficient ( $C_d$ ) and energy dissipation of flow are investigated. The QCSS consists of a quarter-circular as the crest and a stepped chute. The  $C_d$  of QCSS varies between 0.9 and 1.4 considering the range of relative upstream head ( $h_{up}/R$ ) between 0.1 and 1.0, but at the same range of  $h_{up}/R$ , the  $C_d$  of the broad crested stepped spillway (BCSS) changes between 0.9 and 1.0. The ratio of energy dissipation changes between 30% and 98% for both hydraulic models. Comparing the performance of QCSS and BCSS shows that in the skimming flow condition, the  $C_d$  of QCSS is about 30% more than the BCSS. Whereas, in the same flow regime, the energy dissipation of the flow on QCSS is a bit less than the BCSS about 10%.

**Keywords:** Circular Crested Weir; Cascade Weir; Discharge Coefficient; Flow Measurement; Energy Dissipation.

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### Introduction

Spillways are common structures used for flood evacuation systems (FES) in dam projects. The main task of spillways is removing the excess flow and transferring it downstream under safe conditions. The hydraulic efficiency of spillways is very important especially when they are considered as parts of the FES in the earthen dam (Parsaie et al., 2018). Because if their hydraulic performances regarding discharge capacity and safe transferring of flow from the reservoir to the downstream river were not satisfied, the overtopping phenomena may occur (Dehdar-Behbahani and Parsaie, 2016). Therefore, the performance of the spillways should be revised. Based on the governing equation on the discharge capacity of weirs, scholars have tried to propose several ways to improve their discharge capacity. The discharge capacity of weirs is proportional to the upstream head, length of the crest, and discharge coefficient (Bos,

1967). To increase the upstream head, decreasing the elevation of crest or increasing the elevation of embankments may be considered. This approach imposes a high cost of executive operation. To improve the discharge capacity of weirs regarding increasing the length of the crest, nonlinear weirs such as labyrinth and oblique weirs have been proposed (Parsaie and Haghiabi, 2017 & Parvaneh et al., 2016). The main weakness of labyrinth weirs is related to their discharge coefficient, which is dramatically decreased by increasing the upstream head of the flow (Crookston, 2012). The development of weirs with a high discharge coefficient is the most rational approach to improve the discharge capacity. In this regard, circular crested weirs have been proposed as weirs with a high discharge coefficient (Shabanlou and Khorami, 2013, Shabanlou, 2013, Kabiri-Samani and Bagheri, 2014, Schmocker, 2011 & Haghiabi, 2018). As stated above, safe transferring of flow from

the reservoir to downstream is another point assessed regarding the spillway performance. As the flow passes over the crest, the velocity of flow is increased. Increasing the flow velocity is associated with the reduction of the flow pressure. Increasing the flow velocity and decreasing the flow pressure result in cavitation. The high energy of flow passed through the chute of spillways may cause scouring at the toe of spillways (Dargahi, 2013 & Haghiabi, 2017). To remove the probability of occurrence of cavitation, using the deflectors and for minimizing the scouring downstream of spillways, using the energy dissipators has been proposed. On the other hand, using the stilling basin and flip buckets as energy dissipators have been proposed. Attention to these two hazards welcomed the investigators to focus on the stepped spillways (Frizell, 2013 & Parsaie, 2016). Usually, the stepped spillways consist of a broad crest (some time ogee profile) attached to a stepped chute. Stepped spillways dramatically decrease the potential of occurrences of cavitation, highly flow aeration, and significantly dissipate the energy of flow (Chatila and Jurdi, 2004, Christodoulou, 1993, Sorensen, 1985 & Zare and Doering, 2012). The main studies on stepped spillways have been focused on the mechanism of energy dissipation of flow and the effects of geometries of steps on their hydraulic efficiency in regard the head loss. On this point, Felder and Chanson (Felder and Chanson, 2011). studied the impact of non-uniformity in the height of steps on energy dissipation and flow regime. They stated that the effect of non-uniformity of

step height on the increasing energy dissipation is negligible and creates instability in the flow pattern. Felder and Chanson (Felder and Chanson, 2014). investigated the influence of pooled steps on the energy dissipation of flow on stepped spillways. They demonstrated that the formation of pooling on steps increases the energy dissipation by about 30 percent. Gonzalez, Takahashi and Chanson (Gonzalez et al., 2008). declared the rough step on the flow pattern over stepped spillways. They found that the roughness of steps has no significant effect on changing the flow regime in all three known flow regimes. Reviewing literature shows that stepped spillway is a rational approach to remove the cavitation and dissipate the energy of the flow. However, its main disadvantage is the corresponding discharge coefficient. Therefore, in this study, a new form of stepped spillway called quarter-circular crested stepped spillway is introduced and its hydraulic properties are investigated.

#### Materials and Methods

The sketch of the QCSS is shown in Figure (1). As shown in this figure, the QCSS consists of a quarter-circular crest and a stepped chute. The curvature of the crest is known with radius ( $R$ ). To compare the performance of the QCSS with the BCSS, the length of the crest of BCSS was chosen equal to the radius of the crest of QCSS. In the following section, using dimensional analysis, involved parameters on the  $C_d$  of the QCSS and the BCSS are derived. Then, their capability of energy dissipation is defined.

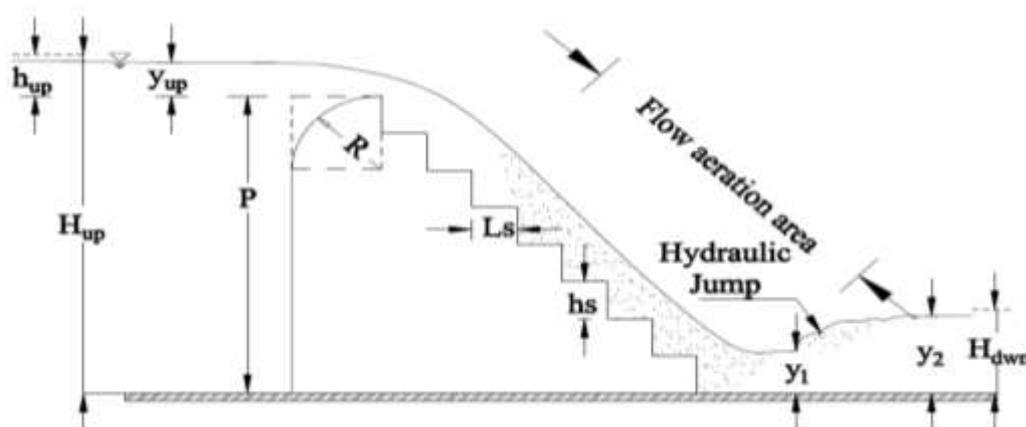


Fig. 1- The Sketch of a quarter-circular crested stepped spillway

### Discharge Coefficient

The discharge capacity is essential to design the structure of spillways. The governing equation on the discharge capacity of weirs is presented in Equation (1) (Haghiabi *et al.*, 2018). As can be found from this equation, the discharge capacity is directly related to the discharge coefficient. The  $C_d$  of QCSS is proportional to its hydraulic and geometric parameters. The most involved parameters on the  $C_d$  are given in Equation (2).

$$q = \frac{2}{3} C_d h_{up} \sqrt{\frac{2}{3} g h_{up}} \quad (1)$$

$$C_d = f(R, h_{up}, P, h_s, L_s, V, g, \rho) \quad (2)$$

where  $R$  is the radius and width of the crest,  $h_{up}$  is the head of flow over the crest,  $P$  is the dam height,  $h_s$  and  $L_s$  are the height and length of steps, respectively,  $\rho$  is the water density,  $g$  is the gradational acceleration, and  $V$  is the velocity of approached flow. Using the Buckingham  $\Pi$  theorem as a dimensional analysis technique, the involved dimensionless parameters on the  $C_d$  are derived and are given in Equation (3). It is notable that  $\rho, V, R$  are considered repetitive parameters and  $N$  is the number of steps.

$$\Pi_1(P) = \rho R (P = N \times h_s) = \frac{P}{R} \Rightarrow \Pi_1(P) = N \frac{h_s}{R}$$

$$\Pi_2(h_s) = \frac{h_s}{R}$$

$$\Pi_1(P) = N \Pi_2(h_s)$$

$$\Pi_3(L_s) = \frac{L_s}{R}$$

$$\Pi_4(g) = \frac{Rg}{V^2} \Rightarrow \sqrt{\frac{1}{\Pi(g)}} = \frac{V}{\sqrt{Rg}} = Fr$$

$$\Pi_2(h_s) \times \frac{1}{\Pi_3(L_s)} = \frac{h_s}{R} \times \frac{R}{L_s} = \frac{h_s}{L_s} = s$$

$$C_d = f\left(\frac{h_{up}}{R}, s, Fr, N\right) \quad (3)$$

where the  $Fr$  is Froude Number,  $N$  is the number of steps,  $s$  is the slope of stepped chute, and  $h_{up}/R$  is the ratio of flow head over the crest to the crest radius (relative upstream head).

### Energy Dissipation

Energy dissipation of the flow passing over the stepped spillway is estimated by applying the Bernoulli equation upstream and downstream of the stepped spillway structure. The total upstream and downstream energies of flow in a rectangular channel are defined as  $H_{up}$  and  $H_{dwn}$ , respectively (Eq. 4 & Eq. 5).

$$H_{up} = P + y_{up} + \frac{V_0^2}{2g} = P + y_{up} + \frac{q^2}{2g(P + y_{up})^2} \quad (4)$$

$$H_{dwn} = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2gy_1^2} \quad (5)$$

Where,  $q$  is the discharge per unit width of stepped spillways,  $y_{up}$  and  $y_1$  are the depths of flow upstream and downstream. The total ratio of head loss (EDR) is evaluated using Equation (6).

$$EDR = \frac{\Delta H}{H_{up}} = \frac{H_{up} - H_{dwn}}{H_{up}} = \left(1 - \frac{H_{dwn}}{H_{up}}\right) \times 100 \quad (6)$$

The involved geometric and hydraulic parameters on the EDR are given in Equation (7).

$$\frac{\Delta H}{H_{up}} = f(R, h_s, L_s, P, V, H_{up}, g, \rho) \quad (7)$$

where  $y_{up}$  is the depth of flow over the stepped spillway. Using the Buckingham  $\Pi$  theorem, the most important parameters of energy dissipation are derived and are given in Equation (8). It is notable that  $\rho, v, R$  are considered repetitive parameters and  $N$  is the number of steps.

$$\Pi_1(P) = \rho R (P)$$

$$\Pi_2(h_s) = \frac{h_s}{R}$$

$$\Pi_1(P) = N \frac{h_s}{R} = N \times \Pi_2(h_s)$$

$$\begin{aligned}
\Pi_3(L_s) &= \frac{L_s}{R} \\
\Pi(h_s) \times \frac{1}{\Pi(L_s)} &= \frac{h_s}{L_s} = S \\
\Pi_4(H_{up}) &= \frac{H_{up}}{R} \xrightarrow{H_{up}=1.5y_c} \Pi(H_{up}) = \frac{y_c}{R} \\
\Pi_5(g) &= \frac{gR}{V^2} = \frac{1}{\Pi(g)} = \frac{V^2}{gR} = \frac{v=q/y_c}{y_c^2 g} \rightarrow \frac{q^2}{y_c^2 g (R)} \\
\frac{1}{\Pi(H_{up})} \times \Pi_5(g) &= \frac{R}{y_c} \times \frac{q^2}{y_c^2 g R} = \frac{q^2}{y_c^3 g} = \frac{g=q^2/y_c^3}{g} \rightarrow \frac{g}{g} = \frac{1}{N} \\
\frac{\Delta H}{H_{up}} &= f\left(\frac{y_c}{R}, S, N\right)
\end{aligned} \tag{8}$$

where  $y_c/R$  is the ratio of critical depth ( $y_c$ ) of flow to the radius of crest.

### Experimental Setups

In order to hydraulically evaluate the plan of the proposed QCSS model, a number of laboratory models were constructed. The geometric details of the models including height, crest radius, size, and number of steps are given in Table (1). The models were made of iron sheets and their surface was covered with the epoxy blue paint. Models were installed in a two-part channel. The length of channel was 12 m, the width was 0.6 m, and the depths of the first and second parts were 0.98 m and 0.5 m, respectively. The longitudinal slope of the channel was 0.0001. The models were made of two separate and connectable parts; the crest and the stepped chute. The measurements were performed in such a way that first, a stepped chute was installed in the channel, and then, wide-edged and quarter-circular crests with radii as mentioned in Table 1 were installed on the

model. The models were installed in such a way that firstly, one of the stepped chute was installed in the channel, and then, the broad and quarter circular crest was installed on it, separately. Thus, for a stepped chute, all of the crests could be examined. The water levels at upstream and downstream of spillway were measured with a point gage and an accuracy of 0.1 mm. The flow discharge was measured by a triangular weir installed at the end of the channel. The velocity of flow in the vertical direction along the crest was measured by Micro-Molina. A pivot weir installed between the stepped chute and triangular weir was used to control the hydraulic jump.

### Results and Discussion

In this section, the results obtained from laboratory measurements of hydraulic properties of QCSS and BCSS are presented. Among the hydraulic properties, flow velocity distribution, discharge coefficient, and energy dissipation are investigated. The profile of flow velocity was measured at the five stations on the crest. The first station was considered at the beginning of crest and at each 0.05m, the profile of flow velocity was recorded. The vertical profile of velocity on the crest of both models (QCSS and BCSS) for depths of flow (hup) 0.05, 0.08, and 0.1m are shown in Figure (2). As shown in this figure, at the same upstream flow depth over the crest, the maximum velocity of BCSS for all tests is more than the maximum velocity of flow over the QCSS. Whereas, at the same values of upstream flow depth, the flow depth over the crest of QCSS is more than the BCSS. The difference between their maximum velocities at the beginning of the crest is very high, while at the end of the crest, the difference is small.

**Table 1- A summary of properties of laboratory models of stepped spillways**

Model	R(m)	hs(m)	Ls(m)	N	Q(m <sup>3</sup> /s)
Model 1	0.1	0.10			
		0.05			
Model 2	0.15	0.10			
		0.05			
Model 3	0.2	0.10			
		0.05			

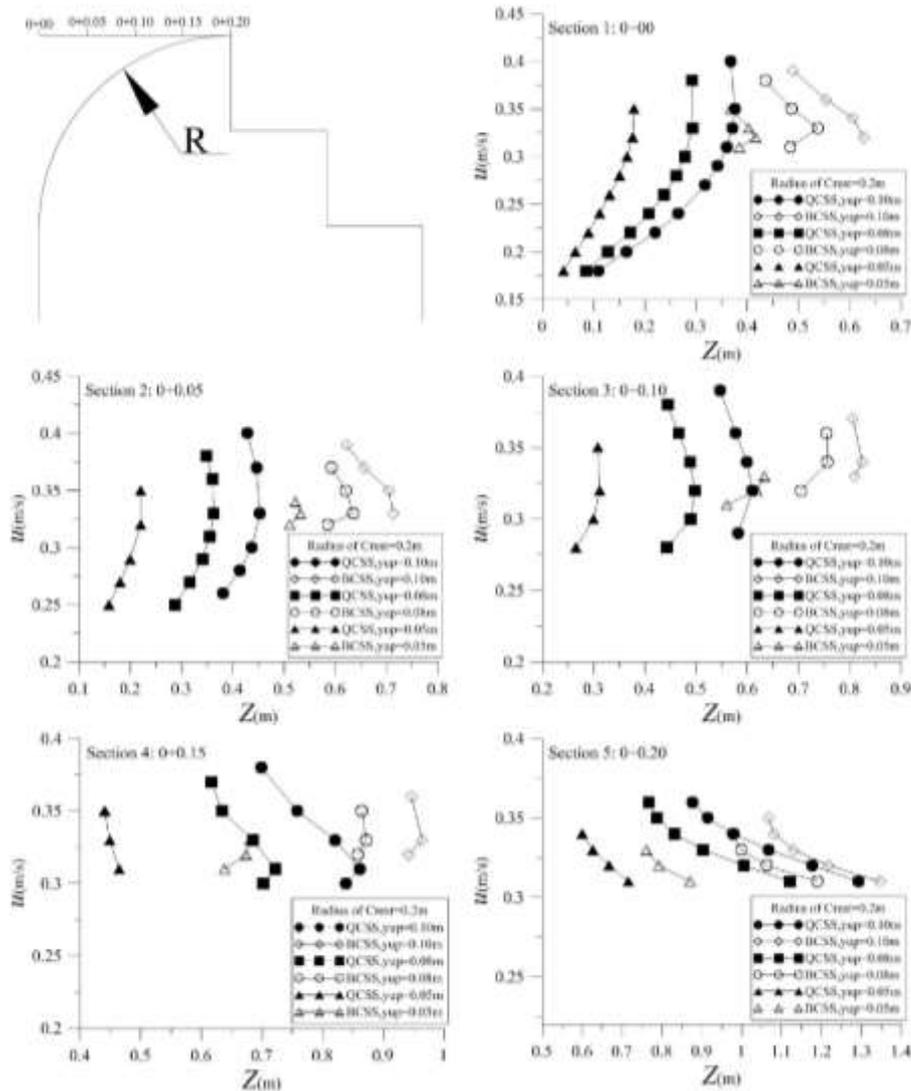


Fig.2- Flow velocity distribution on the crest of QCSS

**Discharge Coefficient**

In this section, the discharge coefficient of QCSS is assessed and then a comparison with the BCSS is conducted. To this end, the stage-discharge relations of BCSS and QCSS are plotted in Figure (3). In this figure, the values of discharge are plotted versus the upstream head. Reviewing this figure shows that by increasing the upstream head, the

discharge capacity of both models are increased, however, for values less than 0.04, the  $C_d$  of both models are close together, and for values more than 0.05, for the upstream head, the discharge capacity of QCSS is more than the BCSS. According to this figure, it is found that by increasing the upstream head, the intensity of increasing the discharge capacity of QCSS is more than the BCSS.

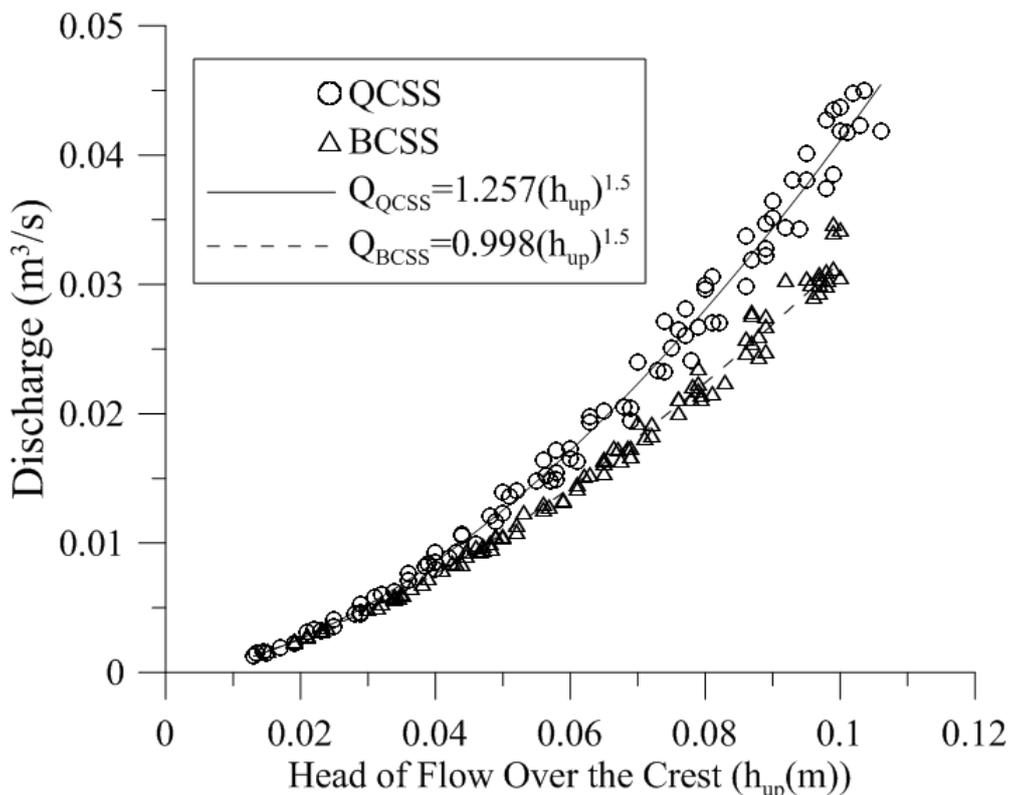


Fig. 3- The Stage-Discharge relations of BCSS and QCSS

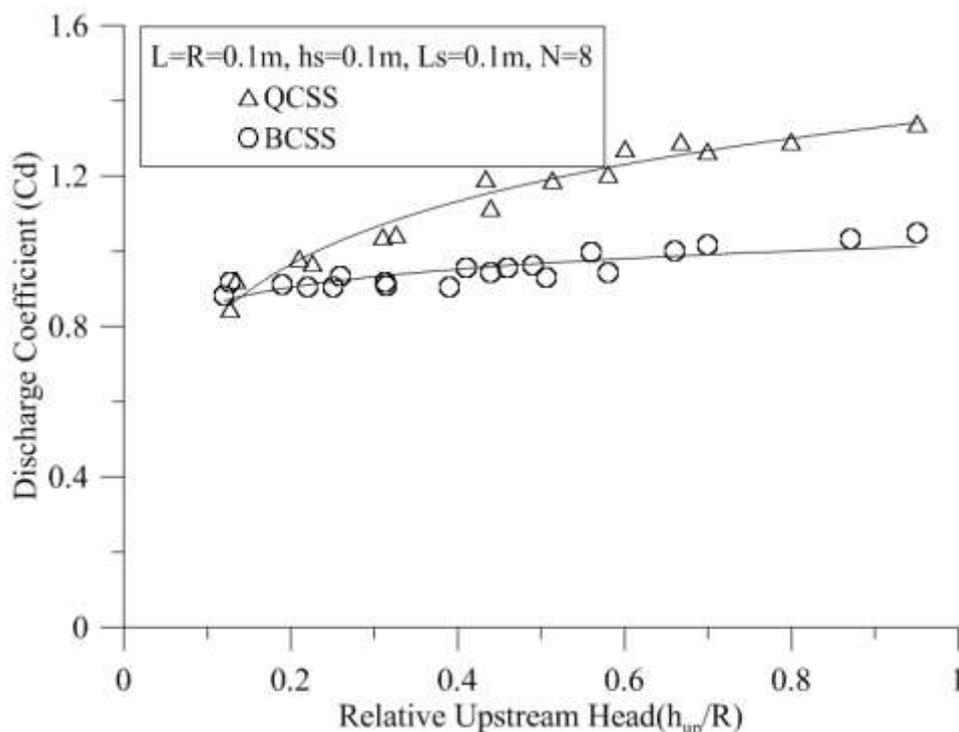


Fig. 4- The values of discharge coefficients of QCSS and BCSS

As presented in the materials and methods section, the involved parameters on  $C_d$  are the relative upstream head ( $h_{up}/R$ ), the slope of the downstream stepped chute ( $S$ ), values

of curvature of the crest ( $R$ ), Froude number ( $Fr$ ), and the number of steps. The values of the  $C_d$  of QCSS and BCSS are given in Figure (4).

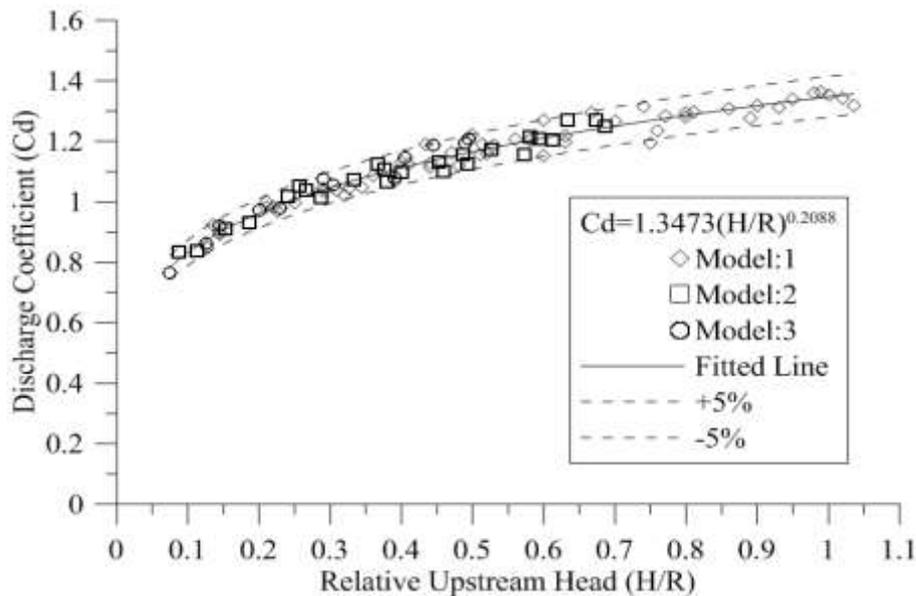


Fig. 5- The effect of downstream slope on Cd of the QCSS versus relative upstream head

As shown in this figure, the Cd of QCSS varies between 0.9 and 1.4, considering the values of  $h_{up}/R$  ranging from 0.1 to 1.0; whereas, the Cd of BCSS at the same value of  $h_{up}/R$  changes between 0.8 and 0.9. This figure shows that in a high value of upstream head ( $h_{up}/R=1.0$ ), the Cd of QCSS is more than the Cd of BCSS about 30%. The most important reason for increasing the Cd of QCSS in comparison with BCSS is the curve of the crest that helps remove the separation region which is created at the beginning of the edge of broad the crest. In general, by increasing the upstream head, the Cd of QCSS and also its intensity is more than the Cd of BCSS; this means that the efficiency of QCSS is increased by increasing the upstream head and it is more reliable. For values less than 0.4 of  $h_{up}/R$ , the intensity of increasing the Cd of QCSS is significantly more than the intensity of increasing the Cd of BCSS, while by increasing the upstream head ( $h_{up}/R$ ) to more than 0.4, the intensity of increasing the Cd of QCSS and BCSS is decreased. To assess the effect of the slope of the chute, three slopes ( $S=1 V:1 H$ , 1:1.5, 1:2) are considered. In other words, at the same range of discharge and height of steps ( $h_s=0.10m$ ), three values for the length of steps (0.10 m, 0.15 m, and 0.20 m) were considered. The values of Cd for different values of QCSS models are shown in Figure (5). By reviewing this figure, it is found that the geometry properties of the downstream

part have no significant effect on the Cd. The results obtained are also matched by the hydraulic of supercritical flow governing the spillways. According to the supercritical flow concept, the control section is located at the upstream and downstream hydraulic conditions do not affect the upstream.

### Energy Dissipation

The ability of FES in terms of energy dissipation of passed flow is another point in evaluating the safety of dams. As stated in the literature, using stepped spillways is a rational decision in terms of energy dissipation, which removes the potential of cavitation occurrence. In this part, the performance of QCSS regarding energy dissipation is investigated and then its performance is compared with the BCSS. As stated in the materials and methods section, the dimensionless geometrical parameter of steps ( $S=hs/Ls$ ), the relative critical flow depth ( $yc/hs$ ) and the number of steps ( $N$ ) are involved in the performance of stepped spillways in terms of energy dissipation. To this end, the EDR is plotted versus these considering parameters. The energy dissipation of flow over QCSS is shown in Figure (6-a). In this figure, the values of EDR are plotted versus the  $yc/hs$  by considering the size of steps. As shown in this figure, the EDR changes between 0.30 and 0.98 regarding the variation of  $yc/hs$  between 0.1 and 1.4. The intensity of variation of EDR for

values of  $yc/hs$  less than the 0.2 is much more than the values more than 0.2(for  $yc/hs$ ). In this figure, the effect of reducing the size of steps is shown as well. As shown, by reducing the size of steps, the performance of stepped spillways in terms of energy dissipation is decreased about 50%. Due to the formation of the skimming flow regime, the effect of increasing the length of steps (decreasing the slope of downstream stepped chute) on EDR is decreased. In the low values of  $yc/hs$  (less than 0.6) and by increasing the length of steps from 0.10 m to 0.20 m ( $hs=0.10$  m), the performance of stepped spillway is increased by about 25%. This is

due to a delay in the change of flow pattern from napped to skimming flow and increasing the flow resistance. The performance of QCSS and BCSS is shown in Figure (6-b). As shown in this figure, the performance of QCSS regarding the energy dissipation is a bit less than BCSS. The maximum difference between the EDR of QCSS and BCSS is about 10%. Reviewing Figure (6-b) shows that by scrolling the flow regime to skimming flow, the intensity of the effect of step size on energy dissipation is decreased.

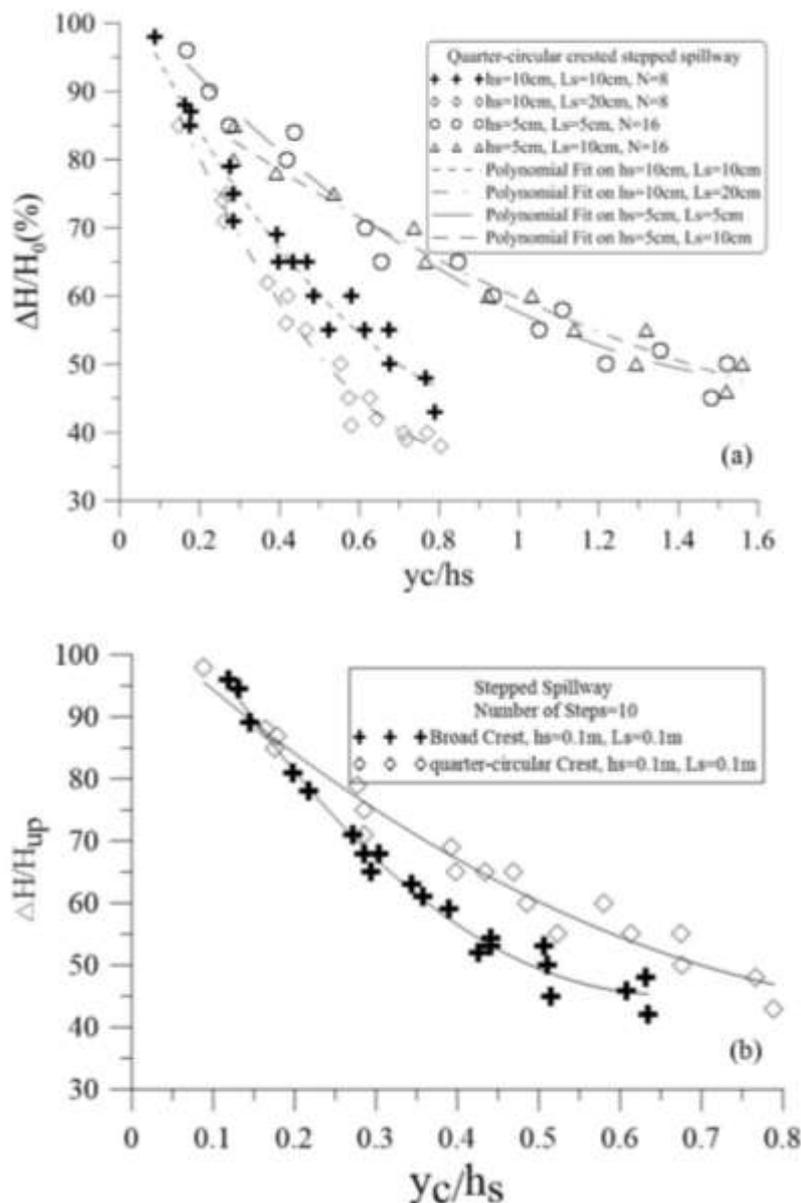


Fig. 6- The values of energy dissipation of flow on QCSS (a) and BCSS (b)

## Conclusion

Stepped spillways are common structures used for the energy dissipation of flow to remove the potential of the occurrence of cavitation. This structure can be used as the flood evacuation system in dam projects. To improve the hydraulic efficiency of stepped spillways in FES, firstly, improving their discharge capacity should be considered. The discharge capacity of such FES is one of the most important factors in evaluating dam safety against the occurrence of PMF. To this end, using weirs with high discharge capacity for the crest of stepped spillway is a rational approach. In the present study, the QCSS was introduced and its hydraulic properties including Cd and EDR were investigated. The quarter-circular crest was used because this type of weir has a high discharge coefficient and by increasing the upstream head (flood condition), its Cd is increased. This means that the performance of this structure is more reliable in critical situations (flooding conditions). Results indicated that

the discharge coefficient of QCSS varies between 0.9 and 1.4 and especially in the skimming flow condition, it is about 30% more than the BCSS. The high discharge coefficient of the quarter-circular crest results in the passage of more discharge compared to the broad crest at the same upstream head. The QCSS can dissipate the energy of flow between 98 (in the napped flow) and 30 (in the skimming flow) percent. The head loss of flow over the QCSS is a bit less than the BCSS.

## Ethical Statements

**Conflict of Interest:** The authors declare that they have no conflict of interest.

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**Ethical approval:** This article does not contain any studies with human participants or animals performed by any of the authors.

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