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Prediction of Transverse Shear Stress in a Rectangular Channel Using Shannon Entropy and Support Vector Regression

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Abstract

In open channel flow, determining the boundary shear stress and its distribution over the wetted perimeter is a significant problem. The shear stress distribution (SSD) is primarily affected by secondary flows, sediment transport rate, erosion or sedimentation, and geometry of the channels. The presented research uses Shannon entropy and support vector regression (SVR) approach to predict the SSD in rectangular channels (RCs). First, the entropy technique proposed by Sterling and Knight, (2002) is used to construct the probability density function of transverse SSD, and the constant coefficients of density are obtained by comparing experimental results in various aspect ratios. Second, to estimate the transverse SSD in a smooth RC, SVR methods have been used. According to the results of the sensitivity analysis, the aspect ratio B/H is the most essential parameter for SSD estimation. The SVR model performed better when the (b/B), (z/H), and (B/H) parameters were also used as input. For the aspect ratios (B/H) 2.86, 4.51, 7.14, and 13.95, the SVR model, with an average MAE of 0.044 in bed and 0.053 in wall, gives higher accuracy than the Shannon entropy, which has an average MAE of 0.062 in bed and 0.073 in wall for all flow depths. The Shannon entropy overestimates shear stress as compared to SVR. As a result, the costs of construction of channels may be significant.

Introduction

In open channel flow, determining the boundary shear stress and its distribution over the wetted perimeter is a significant problem. The SSD is primarily affected by secondary flows, sediment transport rate, erosion or sedimentation, and geometry of the channels. The SSD and flow resistance in simple and compound channels with smooth and rough surfaces were studied by many scholars Kartha and Leutheusser (1970), Yang and Lim (1997), Knight and Patel (1985a), Knight and Patel (1985b), Knight et al. (1994), Sheikh Khozani and Bonakdari (2016), Sheikh Khozani and Bonakdari (2018) and Lashkar-Ara et al. (2021).

It is plausible that the transverse SSD in wide and broad channels is not uniform. Kartha and Leutheusser (1970) conducted a series of experiments on the determination of SSD to design stable alluvial channels by tensile forces. Experiments were performed in a rectangular laboratory flume with smooth-wall in aspect ratios between 1 and 12.5. They measured shear stress by Preston tube. For calibration of the Preston tube, the indirect method was applied, and the law of the wall and velocity logarithmic distribution was used. At that time, they indicated that none of the available analytical techniques could calculate the shear stress for a proper design of alluvial channels.

Lashkar-Ara and Fatahi (2020) have been measuring transverse SSD in rectangular open channels bed and wall using the Preston tube with an optimal diameter. The study's results include two dimensionless relationships for estimating local shear stress in the bed and wall. The aspect ratio *B/H*, as well as the bed relative coordinates *b/B* in cross-section and *z/H* in sidewall, determine these relationships. The study found that the aspect ratio *B/H* has a significant impact on the dimensionless bed SSD. Ardiçlioğlu et al. (2006) did an experimental investigation for the SSD in a fully developed boundary layer area in both smooth and rough surfaces throughout the whole length of the cross-section of an RC. They conducted 48 tests by measuring flow velocity on both smooth and rough surfaces.

The logarithmic distribution of flow velocity was used to calculate mean transverse shear stresses for various aspect ratios *B/H* ranging from 4.2 to 21.6 and Froude numbers ranging from 0.12 to 1.23. Since experimental methods are difficult and time-consuming to calculate the SSD in channels, soft computing methods are suggested. Cobaner et al. (2010) utilized Artificial Neural Networks (ANN) to model boundary shear stress in a smooth RC.

Sheikh Khozani et al. (2018) employed a support vector machine (SVM) to estimate shear stress in a rough RC. To obtain new relationships for the velocity and SSD in open channels, Chiu (1987) proposed new hydraulic principles of maximum entropy and chance. The Shannon's entropy, which is the basis behind such analysis like the maximum entropy concept was further discussed by Chiu (1991) and similar studies by Shuyou and Knight (1996) have been used by Araújo and Chaudhry (1998) and Sterling and Knight (2002) to estimate open channel shear stress. Based on the work of Chiu (1991), Shuyou and Knight (1996), Araújo and Chaudhry (1998), and Sterling and Knight (2002) employed Shannon entropy theory to estimate SSD in open channels.

Sterling and Knight (2002) proposed a novel method for estimating SSD in open channels with a circular cross section. One pitfall of that study is the limitation that the model has to cover all extent of hydraulic behaviors of a channel, which in turn could overshadow the model's reliability. One reason is the difficulty behind the choice of parameter assumptions and the resulting sensitivity to estimate those parameters.

In this research, the efficiency of the Shannon entropy method in estimating SSD in a smooth RC has been evaluated. For this aim, first, the method presented by Sterling and Knight (2002) is implemented to derive the probability density function of shear stress transverse distribution, the constant coefficients of density are determined by examining the findings of the Lashkar-Ara and Fatahi (2020). In the second step, the support vector regression (SVR) function is investigated in the SSD estimation. Finally, the outcomes of these two approaches are compared to each other as well as the Lashkar-Ara and Fatahi (2020) experimental results.

The aim of the presented study was to use the entropy method of Shannon to estimate the SSD in a smooth RC. The outcome of the Shannon entropy method was compared with the SVR model and the experimental outcomes of Lashkar-Ara and Fatahi (2020). The result of the literature review shows that no document has been published on the use of Shannon entropy and SVR in the estimation of SSD in smooth RC's.

Methodology

Data Collection

Data were obtained from Lashkar-Ara and Fatahi (2020) experiments, which were conducted in 10-meter length flume with 60 cm width and 70 cm height. These experiments were carried out at the hydraulic laboratory of Jundi-Shapur University of Technology, Dezful, Iran. The flow rate ranged from 11.06 to 102.38 liters per second for all measurements. Flow rate variations cause changes in water depth from 0.043 m to 0.21 m, as well as changes in the aspect ratio (*B/H*) ranging from 2.86 to 13.95. Pressure transmitter device with a capacity of 0.2 bar and a 50 Hz measuring frequency were used to measure and evaluate the values of total pressure and static difference in various of *B/H*. A weir was added at the end of the flume to obtain uniform flow conditions. The experimental flume is shown in Fig. (1).

Based on previous experimental and field study results, the effective criterion for measuring the SSD throughout the wet perimeter of a channel may be described as follows:

$$f_1(\bar{\tau}_w, \varrho, \upsilon, g, \mathrm{V}, \mathrm{H}, \mathrm{S}_w, S_o, B, z, K_s) = 0 \qquad (1)$$

$$f_2(\bar{\tau}_b, \varrho, \upsilon, g, \mathrm{V}, \mathrm{H}, \mathrm{S}_w, S_o, B, b, K_s) = 0 \qquad (2)$$

where $\bar{\tau}_b$ is the mean bed shear stress, $\bar{\tau}_w$ is the mean wall shear stress, υ is the kinematic viscosity, ρ is the density, *V* is the velocity of flow, *g* is the gravity acceleration, *H* is the depth of flow, *B* is the bed width of flume, K_s is the height of roughness and S_w is the slope of water surface. As demonstrated in Eqs. (3) and (4), the Buckingham- π theorem was utilized to derive dimensionless parameters for wall and bed shear stress.

$$f_3\left(\frac{\upsilon}{VH}, \frac{K_s}{H}, \frac{gH}{V^2}, \frac{B}{H}, \frac{z}{H}, \frac{\bar{\tau}_w}{\varrho g H S_w}\right) = 0 \qquad (3)$$

$$f_4\left(\frac{\upsilon}{VH}, \frac{K_s}{H}, \frac{gH}{V^2}, \frac{B}{H}, \frac{b}{B}, \frac{\bar{\tau}_b}{\varrho g H S_w}\right) = 0 \qquad (4)$$

Equations (3) and (4) may be rewritten as (5) and (6) in the case of smooth channels:

$$\frac{\bar{\tau}_w}{\varrho g H S_w} = f_5\left(\mathrm{Re}, \mathrm{Fr}^2, \frac{B}{H}, \frac{z}{H}\right) \qquad (5)$$

$$\frac{\bar{\tau}_b}{\varrho g H S_w} = f_6\left(\mathrm{Re}, \mathrm{Fr}^2, \frac{B}{H}, \frac{b}{B}\right) \qquad (6)$$

Where (Fr) is the Froude number and (Re) is the Reynolds number. In a smooth RC with varying flow depths, 100 data of shear stress on wall τ_w and 160 data of shear stress on bed τ_b from Lashkar-Ara and Fatahi (2020) were selected for Shannon entropy and SVR models assessment. A total of 70% of the data was picked for training and 30% for testing. Table (1) contains a summary of the experiments.

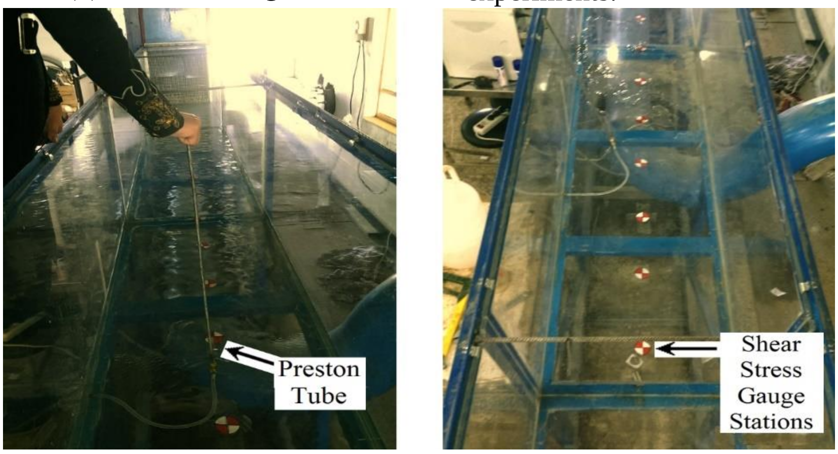

Fig. 1- Experimental setup (Lashkar-Ara and Fatahi, 2020)

Table 1- Summary of the experiment

Parameters	Variable	Min.	Max.	Mean
H (m)	Depth of flow	0.043	0.21	0.0928
B/H	aspect ration	2.86	13.95	7.98
Q (L/s)	Discharge	11.06	102.38	34.795
V (m/s)	Velocity	0.429	0.813	0.568
Fr	Froude number	0.66	0.566	0.618
$\mathrm{Re} \times 10^4$	Reynolds number	6.4	39.87	16.418
Re_*	Shear Reynolds	0.322	0.609	0.426
γHS	Total shear stress	0.442	2.162	0.955

Shannon entropy

The Lagrange coefficient was used by Sterling and Knight (2002) to maximize Shannon entropy and to provide an equation to estimate shear stress:

$$\tau = \frac{1}{\lambda}\left(1 + \left(e^{\lambda\tau_{max}} - 1\right)\frac{y}{L}\right) \quad (7)$$

Where τ is the shear stress of local boundary, τ_{max} is the maximum boundary shear stress, y is the transverse coordinate, L is half of the wet perimeter and the multipliers of the λ which can be calculated as:

$$\lambda = \left(\frac{\tau_{max}\ e^{\lambda\tau_{max}}}{e^{\lambda\tau_{max}} - 1} - \rho gRS\right)^{-1} \quad (8)$$

Where ρ is the mass density, R is the hydraulic radius, g is the gravitational acceleration and, S is the channel's bed slope. This formula was used only in the cross-section of the circle. In a circular channel with a flatbed, the relationships between the wall and the bed must be defined as follows:

$$\tau_w = \frac{1}{\lambda_w}\left(1 + \left(e^{\lambda_w\tau_{max(w)}} - 1\right)\frac{2(y - y_w)}{P_w}\right) y_w \langle y \langle \frac{P_w}{2} \quad (9)$$

$$\frac{1}{\lambda_b}\left(1 + \left(e^{\lambda_b\tau_{max(b)}} - 1\right)\frac{2(y - y_w)}{P_b}\right) \frac{P_w}{2} \langle y \langle \frac{P_b}{2} + y_w \quad (10)$$

Where P is the channel perimeter, and λ are the Lagrange multiplier each corresponding to the channel bed and wall denoted by the subscript of b and w, respectively. It's worth to mentioning that before using the Eqs. (9), (10) and Eq. (7), we have to estimate the mean and maximum shear stresses.

A pair of mean and maximum shear stresses are required to calculate the SSD. For estimating the values of τ_{max} and $\bar{\tau}$, the results of the Lashkar-Ara and Fatahi (2020) studies were applied. They set the flume's bed slope at 9.58×10^{-4}. Aspect ratios of 2.86, 4.51, 5.31, 6.19, 7.14, 7.89, 8.96, 10.71, 12.24, and 13.95 were used to determine the shear stress distributed by the walls and bed. Preston tubes were used to measure the shear stress distribution. Assuming a completely turbulent and subcritical regime among all the experimental data, the best fit equation for τ_{max} and $\bar{\tau}$ separately for wall and bed in aspect ratio 2.89 < *B/H* < 13.95 was fitted.

Equations (11)– (14) demonstrate the relationships between the variables.

$$\frac{\bar{\tau}_w}{\rho gRS} = \frac{2.1007 + 0.0462\left(\frac{B}{H}\right)}{1 + 0.1418\left(\frac{B}{H}\right) + \left(\frac{B}{H}\right)^{-0.0424}} \quad (11)$$

$$\frac{\bar{\tau}_b}{\rho gRS} = \frac{2.0732 - 0.0694\left(\frac{B}{H}\right)}{1 - 0.146\left(\frac{B}{H}\right) + \left(\frac{B}{H}\right)^{-0.1054}} \quad (12)$$

$$\frac{\tau_{max\,w}}{\rho gRS} = \frac{2.5462 + 6.5434\left(\frac{B}{H}\right)}{1 + 6.34\left(\frac{B}{H}\right) + \left(\frac{B}{H}\right)^{-0.1083}} \quad (13)$$

$$\frac{\tau_{max\,b}}{\rho gRS} = \frac{3.157 + 0.8214\left(\frac{B}{H}\right)}{1 + 0.8535\left(\frac{B}{H}\right) + \left(\frac{B}{H}\right)^{-0.1401}} \quad (14)$$

The mean and maximum wall and bed shear stress are $\bar{\tau}_w \& \bar{\tau}_b$ and $\tau_{max\,w} \& \tau_{max\,b}$, respectively. As a result, based on *B/H* and S_o, the transverse SSD for an RC can be estimated.

Using the results of Lashkar-Ara and Fatahi (2020) and determining the values of $\bar{\tau}_w \& \bar{\tau}_b$ using Eqs. 11 to 14, the Lagrange coefficients were calculated by Eqs 9 and 10. The results are summarized in Table (2).

Support Vector Regression analysis

The fundamental basis of SVM has been developed by Vapnik (1998), which receives wide admiration among researchers because of having characteristics of a great empirical performance. This machine learning method works on the identification of a hyper-plane constructed in an infinite-dimensional space that separates two classes in a series of classification. An SVM has been used for classification, regression, or other similar tasks Ebrahimi and Rajaee (2017). Support vector machines have been known by two main categories: support vector classification (SVC) and SVR.

SVM is a method of a learning machine that uses a high dimensional space. This offers predictive functions that are built on a support vector subset. SVR relies only on a sub-set of training data, as the cost function for model construction is not concerned with training points beyond the margin Basak et al. (2007). The efficiency of the generalization of SVR is calculated by the calibration of the kernel function and parameters. C and π are SVR parameters and indicate the constant regularization and constant kernel function which control the complexity of the model prediction (regression), and the kernel function changes the input space dimensionality to perform the regression process more confidently, respectively. Due to its accuracy and reliable efficiency, RBF has become the researcher's option as the kernel function for SVR over the years Suryanarayana et al. (2014). SVM's regression subset, known as SVR, was applied to estimate the transverse SSD in a smooth RC. Therefore, the Radial Basis Function (RBF) is adopted in this study and is expressed as

$$k(x_i, x) = exp(\gamma \|x - x_i\|^2) \quad (15)$$

The estimation precision is determined by collecting of the three parameters of the SVR, namely C, γ, and ε. Using the trial and error method, these parameters are standardized, and results are shown in table (3).

Statistical analysis

The four statistical assessment metrics used to determine the Shannon entropy and SVR models performance are the Maximum Error (ME), Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and Nash–Sutcliffe Efficiency (NSE), which are calculated as follows (Willmott and Matsuura (2005) and McCuen et al. (2006)):

$$\mathrm{ME} = \mathrm{Max}|P_i - O_i| \quad (16)$$

$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N}|P_i - O_i| \quad (17)$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(P_i - O_i)^2}{N}} \quad (18)$$

$$\mathrm{NSE} = 1 - \frac{\sum_{i=1}^{N}(P_i - O_i)^2}{\sum_{i=1}^{N}(O_i - \bar{O})^2} \quad (19)$$

where O_i represents the observed parameter value, P_i represents the projected parameter value, $\bar{O}$ represents the mean observed parameter value, and N represents the number of samples.

Table 2- Summary of the results of Lagrange coefficients in the Shannon entropy model

b/h	Experimental shear stress		Lagrange coefficents	
	$\bar{\tau}_b$	$\bar{\tau}_w$	λ_b	λ_w
2.86	1.435	1.200	1.415	8.262
4.51	1.012	0.846	2.087	11.510
5.31	0.894	0.738	3.099	15.322
6.19	0.761	0.638	3.099	15.322
7.14	0.709	0.556	3.515	12.204
7.89	0.655	0.498	4.540	12.305
8.96	0.603	0.450	5.930	14.716
10.71	0.516	0.372	7.485	16.433
12.24	0.466	0.326	10.213	18.122
13.95	0.409	0.278	13.687	20.985

Table 3- Kernel constant after try and error

Kernel function	Kernel constants		
	C	γ	ε
RBF	100	0.1	1

Results and discussion

SVR modeling

The sensitivity of the SVR model to each input parameter is assessed in this section. Three models were developed for this purpose. SVR models implemented in the present study are described as:

For the bed:

$$SVM\ Model(1): \frac{b}{B}, \frac{B}{H}, Fr, \mathrm{Re}$$

$$SVMModel(2): \frac{b}{B}, \frac{B}{H}, Fr,$$

$$SVMModel(3): \frac{b}{B}, \frac{B}{H},$$

For the wall:

$$SVMModel(1): \frac{z}{H}, \frac{B}{H}, Fr, Re$$

$$SVMModel(2): \frac{z}{H}, \frac{B}{H}, Fr,$$

$$SVMModel(3): \frac{z}{H}, \frac{B}{H},$$

Different models were examined for every channel segment to examine the influence of each input parameter on the SVR model accuracy. The results of the simulation of the bed shear stress showed that the SVR model (1) consists of the input parameters (*b/B*, *B/H*, Fr, Re) had the lowest error (average RMSE = 0.055), and in wall shear stress modeling, SVR model (Model 2) with the inputs *z/H*, *B/H*, Fr had the lowest error (average RMSE=0.064). Among the SVR models with ternary variables of input, Model 3, with *b/B* and *B/H* as input variables with NSE value of 0.942, performed the best in modeling bed shear stress while Model 3 with *z/H* and *B/H* as input variables with NSE value of 0.832, performed the best in modeling wall shear stress. Therefore, *B/H* has a significant impact on the SVR model efficiency and validates the Model 3 performance. As a result of the sensitivity analysis, the Reynolds number can be ignored in Model 2 because the flow condition is fully developed. The statistical results of the SVR model with different input combinations are shown in Table (4).

Table 4- Evaluation results of the SVR model with various input cases

B/H	Input Variable	Bed				Input Variable	Wall			
		ME	MAE	RMSE	NSE		ME	MAE	RMSE	NSE
2.86	*b/B,B/H,* Fr, Re	0.1938	0.0495	0.0787	0.9589	*z/H,B/H,* Fr, Re	0.0617	0.0363	0.0516	0.8021
2.86	*b/B,B/H,* Fr	0.2045	0.0650	0.0885	0.9571	*z/H,B/H,* Fr	0.0343	0.0288	0.0450	0.8416
2.86	*b/B,B/H*	0.1443	0.0690	0.0789	0.9502	*z/H,B/H*	0.0383	0.0278	0.0424	0.8026
4.51	*b/B,B/H,* Fr, Re	0.5270	0.0176	0.0246	0.9951	*z/H,B/H,* Fr, Re	0.0668	0.0493	0.0546	0.9189
4.51	*b/B,B/H,* Fr	0.0619	0.0286	0.0335	0.9944	*z/H,B/H,* Fr	0.1402	0.0442	0.0545	0.9314
4.51	*b/B,B/H*	0.0650	0.0229	0.0301	0.991	*z/H,B/H*	0.1030	0.0388	0.0496	0.9095
7.14	*b/B,B/H,* Fr, Re	0.0489	0.0269	0.0294	0.9916	*z/H,B/H,* Fr, Re	0.0744	0.0614	0.0514	0.9216
7.14	*b/B,B/H,* Fr	0.0451	0.0255	0.0277	0.9922	*z/H,B/H,* Fr	0.0482	0.0501	0.0538	0.893
7.14	*b/B,B/H*	0.0425	0.0243	0.0266	0.9907	*z/H,B/H*	0.1037	0.0376	0.0501	0.8954
13.95	*b/B,B/H,* Fr, Re	0.1205	0.0677	0.0903	0.8318	*z/H,B/H,* Fr, Re	0.0720	0.0612	0.1045	0.6942
13.95	*b/B,B/H,* Fr	0.1219	0.0607	0.0878	0.8287	*z/H,B/H,* Fr	0.2400	0.0942	0.1061	0.7125
13.95	*b/B,B/H*	0.1678	0.0716	0.0978	0.8394	*z/H,B/H*	0.2092	0.1109	0.1246	0.7238
Average Model (1)		0.222	0.0404	0.055	0.944		0.068	0.052	0.065	0.834
Average Model (2)		0.108	0.044	0.059	0.943		0.115	0.054	0.064	0.844
Average Model (3)		0.105	0.047	0.058	0.942		0.113	0.053	0.066	0.832
Total Average		0.145	0.044	0.057	0.943		0.099	0.053	0.065	0.837

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There is no significant difference when Reynolds number (Re) is left out of the input parameters, as indicated in the table. The influence of the Froude number may be ignored because all of the experiments were conducted at subcritical flow conditions; hence, the parameter Fr has been omitted from Model 3. The SVR model's performance was not much improved by omitting the Re and Fr parameters, and the SVR model appears to be sensitive to the *B/H* parameter. It is clear that the *B*/*H* ratio is essential in shear stress prediction, as this parameter plays an important role in the mentioned equations.

Therefore, Model 3 is chosen as the most suitable model for the bed and wall. Figures (2) and (3) show the estimated results of the SVR model plotted as scatter against the experimental data for the dimensionless parameter of bed and wall shear stresses. As seen in these figures, the regression analysis shows the results obtained by the SVR model have almost fitted against the experimental data for both the bed and wall shear stresses of the channel. The result of NSE is higher for the dimensionless bed shear stress (average NSE is 0.942) than dimensionless wall shear stress (average NSE is 0.832), and both models are shown to have better performance than the other SVR models implemented in this study.

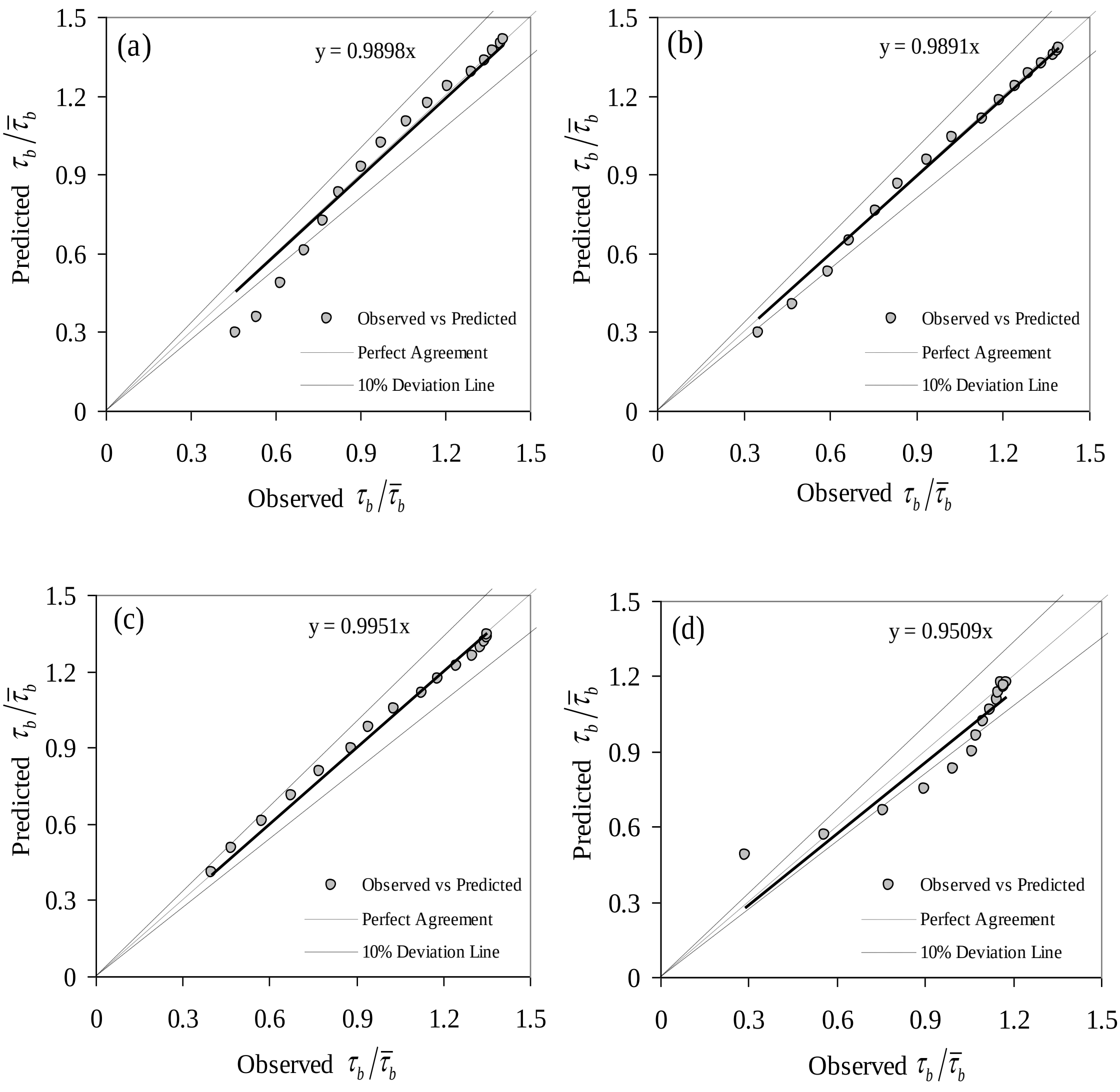

Fig. 2- Comparison SVR result in $\tau_b/\bar{\tau}_b$prediction versus laboratory observations at : (a) *B/H*=2.86, (b) *B/H*=4.51, (c) *B/H*=7.14, and (d) *B/H*=13.95.

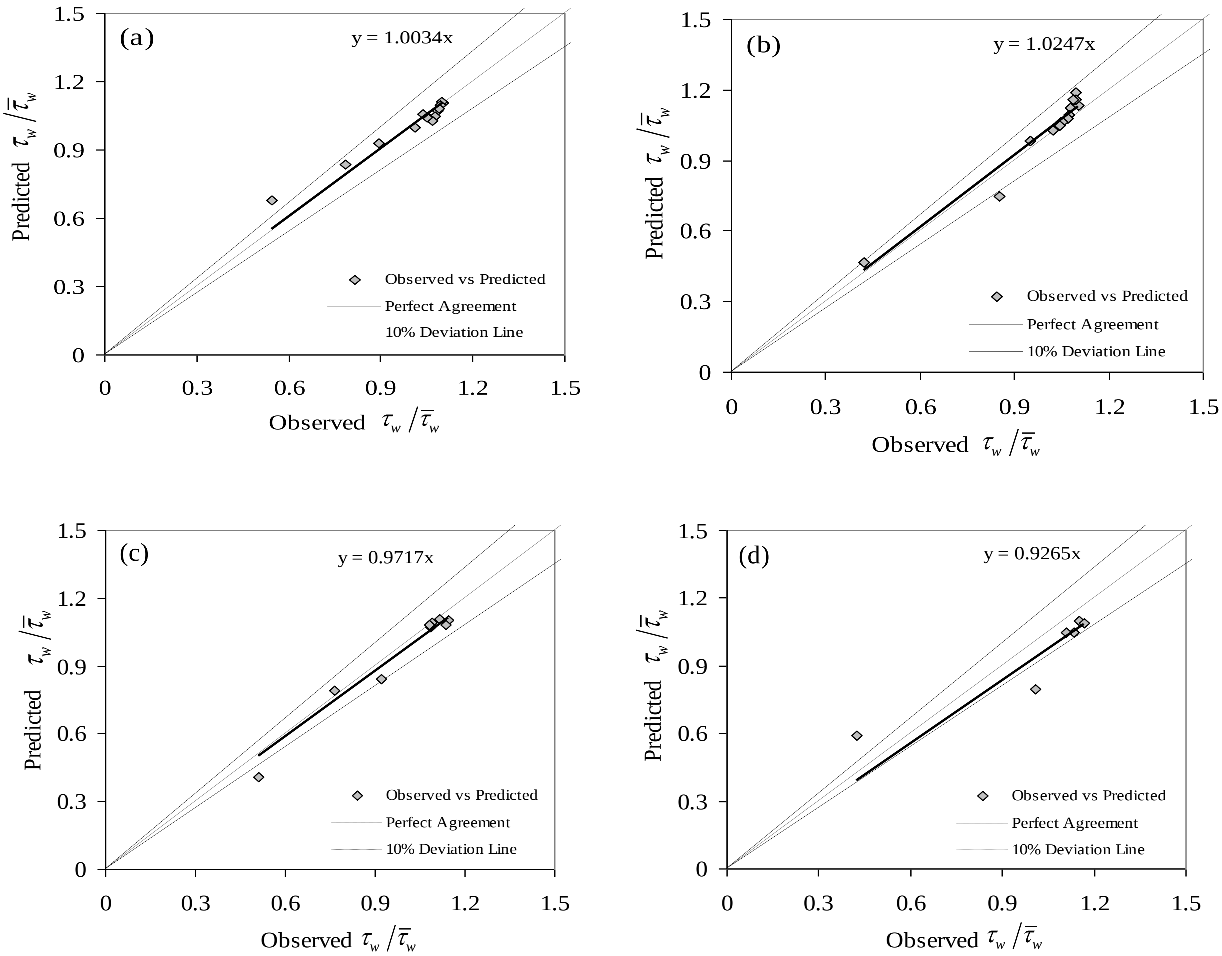

Fig. 3- Comparison SVR result in $\tau_w/\bar{\tau}_w$ prediction versus laboratory observations at : (a) *B/H*=2.86, (b) *B/H*=4.51, (c) *B/H*=7.14, and (d) *B/H*=13.95.

Model comparisons

In this part, the results of the best SVR models and Shannon entropy in predicting shear stress were compared to the observation of Lashkar-Ara and Fatahi (2020). Figures 4 and 5 show the laboratory results and SSD predictions with different models in a smooth RC for *B/H* equal to 2.85, 4.51, 7.14 and 13.95. In addition, Table (4) shows the Shannon entropy model's performance metric for estimating SSD.

All the test data utilized to model SSD using the SVR are well recognized as seen in these statistics. According to the statistical metric, all test functions used to estimate the SSD using the SVR were realized. 70% of total data was used for training and 30% was used for testing the SVR models for SSD estimation. The SVR model predicted bed shear stress better than the Shannon entropy model for all aspect ratios *B/H*, as shown in Figure 4. The SVR estimates the bed SSD better compared to Shannon entropy for *B/H*=2.86, 4.51, 7.14 (Fig. 4(a) to 4(c)) while the Shannon entropy model seems to be better than the SVR for *B/H*=13.95 (Fig. 4(d)). The SVR model predicted wall SSD better than the Shannon entropy model in Figures 5(a), 5(b), and 5(c) for *B/H* = 2.86, 4.51, and 7.14, respectively, while the Shannon entropy model predicted wall SSD more precisely in Figure 5(d). At increasing flow depth, the SVR model predicted bed and wall shear stress better than the Shannon entropy-based model.

When a model predicted higher shear stress values, it's obvious that channel design would be problematic. As a result, using the Shannon entropy technique to channel construction might be challenging. When the SVR model's observations became more precise, it could be utilized to more consistently design stable channels. As for one example, both methods ignore the effect of secondary flows; however, in the case of the SVR model, the results show improved performance. The results showed that the flexibility of the SVR model to estimate SSD is higher than the Shannon entropy and can overestimate the results when faced with a channel's most uncertain behaviors. The bed shear stress values are decreased in the center of the channel (Fig. 5), which varies from other cases. As demonstrated in Figures 2 and 3, the SVR model's fit line is closer to the best fit line than the other models, and its prediction is more accurate with a higher NSE value based on statistical metrics (Tables 4 and 5).

The SSD predictions based on SVR and Shannon entropy models exhibit the same tendency in evaluating the location of peak shear stress as the channel centerline, which is closer to the experimental results.

Table 5- Shannon entropy model statistical outcomes compared to experimental data

B/*H*	Bed				Wall			
	ME	MAE	RMSE	NSE	ME	MAE	RMSE	NSE
2.86	0.187	0.0611	0.0746	0.9332	1.3091	0.0678	0.0777	0.7907
4.51	1.338	0.0560	0.0655	0.9486	1.3091	0.0754	0.0830	0.7899
7.14	1.388	0.0620	0.0705	0.9419	1.660	0.0651	0.0787	0.8423
13.95	1.423	0.0685	0.0740	0.8665	0.9562	0.0846	0.1017	0.8548
Average	1.084	0.062	0.071	0.922	1.308	0.073	0.085	0.819

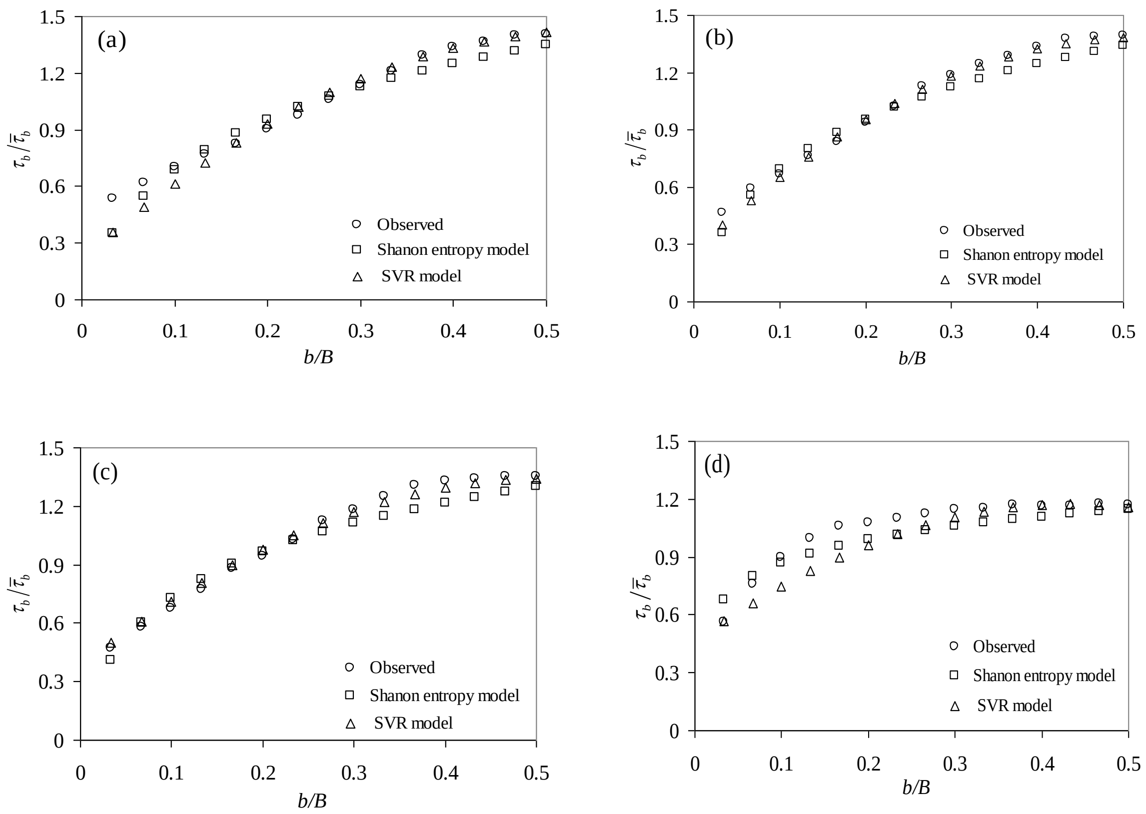

Fig.5- Comparison the prediction of $\tau_b/\bar{\tau}_b$ distribution by Shannon entropy model versus laboratory observations and SVR model at: (a) *B/H*=2.86, (b) *B/H*=4.51, (c) *B/H*=7.14, and (d) *B/H*=13.95.

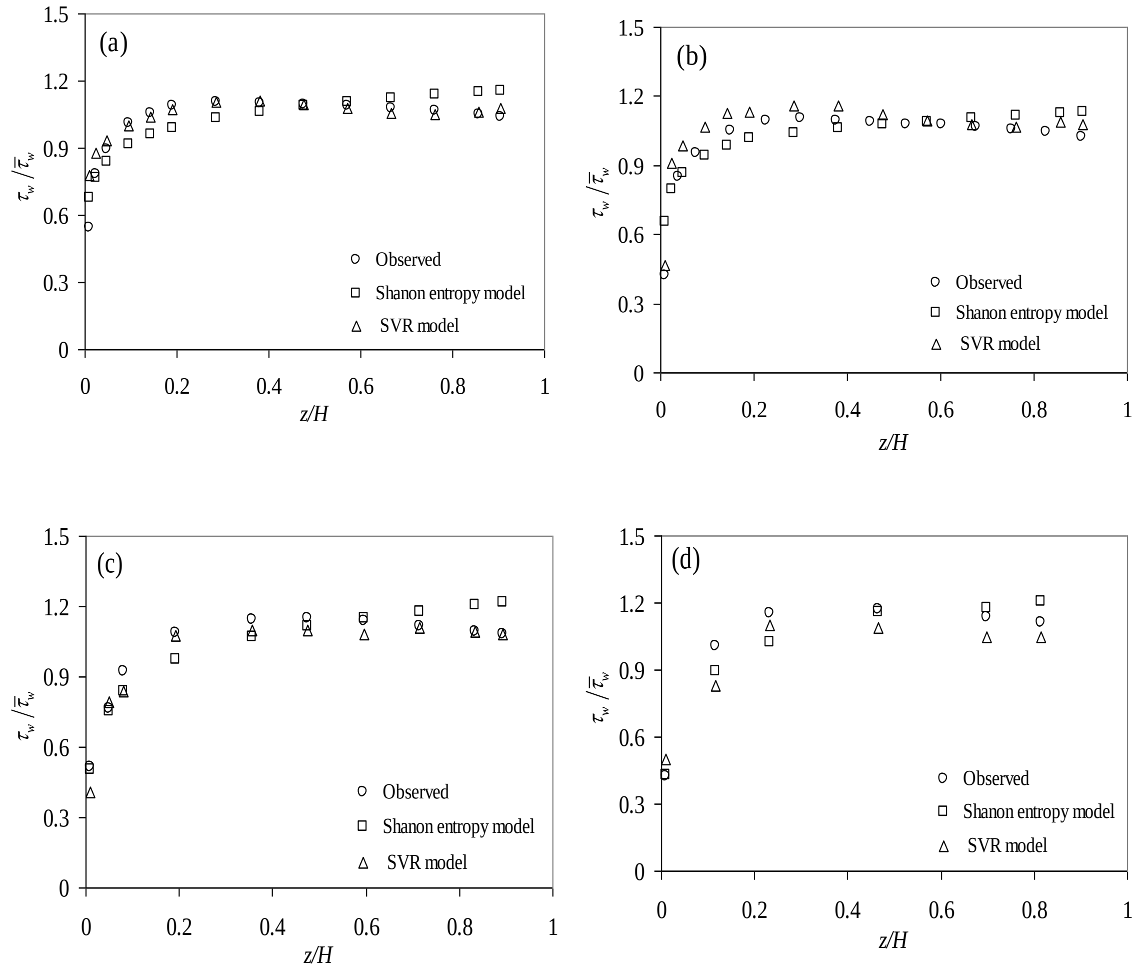

Fig. 6- Comparison the prediction of $\tau_w/\bar{\tau}_w$distribution by Shannon entropy model versus laboratory observations and SVR model at: (a) *B/H*=2.86, (b) *B/H*=4.51, (c) *B/H*=7.14, and (d) *B/H*=13.95.

Conclusions

The determination of SSD in open channels is an essential problem to be solved by engineers. This study examined the use of Shannon entropy and SVR method to predict the SSD in RC's. For this purpose, using the method presented by Sterling and Knight (2002) to derive the probability density function of shear stress transverse distribution, the constant coefficients of density are determined by comparing with the laboratory results of (Lashkar-Ara and Fatahi, 2020). Sensitivity analysis was applied, and three separate SVR models were used to evaluate the effective parameters on the SSD. The outcomes show that B/H is a sensitive parameter in estimating the SSD. Shannon entropy-based formula was compared with the SVR model in estimating SSD. For rising flow depths, both the SVR and the Shannon-based entropy formula provided well accuracy. The SVR, with an average RMSE of 0.057 and NSE of 0.943 performed superior to the Shannon entropy method with the RMSE of 0.074, NSE of 0.922 in bed SSD estimation for the aspect ratios (*B/H*) 2.86, 4.51, 7.14 and 13.95. In wall SSD estimation, also the SVR with the RMSE of 0.065 and NSE of 0.837 outperformed the Shannon entropy with the RMSE of 0.085 and NSE of 0.819 for the all aspect ratios. However, with increasing flow depth, the SVR model measures wall shear stress better than the entropy model. Channel design is known to incur higher costs when a model overestimates shear stress values. As a result, using SVR instead of Shannon entropy to estimate SSD in RC and design stable channels may be less risky and costly to implement.

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