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Numerical investigation of Semicircular Sharp-Crested Weirs

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Abstract

Sharp-crested weirs (SCWs) are widely used hydraulic structures for measuring discharge in small and confined channels. Their hydraulic performance is primarily governed by the relationship between flow head and discharge (H-Q), along with the associated discharge coefficient (C_d). The derivation of the H-Q relationship typically involves the integration of the velocity profile over the flow cross-section. However, for semicircular sharp-crested weirs (SCCWs), this process becomes analytically complex due to the emergence of elliptic integrals. To address this, the present study employed Simpson's numerical integration rule to approximate the relevant elliptic integrals with high accuracy, with a specific focus on determining C_d. A simplified analytical expression for the H-Q relationship was developed and thoroughly validated against an extensive dataset obtained from laboratory experiments. The model exhibited strong agreement with observed data, confirming its predictive capability. Furthermore, dimensional analysis using Buckingham's π-theorem revealed that the discharge coefficient is a function of the dimensionless ratios H/P, H/D, and D/P, where H is the upstream flow head, P is the weir height, and D is the crest diameter. The observed C_d values ranged between 0.95 and 0.97 within a 95% confidence interval for flow conditions where H/P varied from 0.06 to 1.1 and D/P from 0.33 to 3.54. In addition, a nonlinear empirical model was formulated to estimate Cd, achieving a mean absolute percentage error (MAPE) of approximately 7.4%, indicating the robustness and applicability of the proposed methodology for semicircular weir configurations.

Introduction

Weirs are hydraulic structures commonly used in water engineering for various purposes, such as flow measurement, increasing upstream flow depth at water intake structures, and controlling flow rates in open channels. Among the various types of weirs, sharp-crested weirs are especially prevalent for flow measurement in small irrigation canals. These can be designed with different cross-sectional shapes, including rectangular, triangular, trapezoidal, and circular (Bos, 1976). Recently, innovative cross-sectional designs for weirs have been introduced, including full and half-cycle cosine weirs Azimi and Salehi (2022); Salehi et al. (2019), compound weirs Ansari et al., (2019); Zahiri et al., (2014), and combinations with other flow measurement

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structures like gates and orifices (e.g., weir-gate and weir-orifice configurations) (Zeinivand et al. 2024 b & a). According to hydraulic principles, the flow rate over these weirs depends on the upstream flow head and the length of the weir's crest. While the discharge capacity relates linearly to crest length, it follows a power function concerning the upstream flow head (Akan and Iyer 2021). Notably, the accuracy and sensitivity of flow measurements are influenced by this power relationship. For example, the discharge capacity of rectangular weirs is proportional to the upstream head raised to the power of 1.5 ($Q_{rec} \propto h^{1.5}$). In contrast, for triangular weirs, it is proportional to the power of 2.5 ($Q_{trn} \propto h^{2.5}$) (Bos, 1976). However, these relations indicate that triangular weirs offer more precision and sensitivity than rectangular weirs. Although triangular weirs provide greater measurement accuracy, following a power function of 2.5, they have a significantly lower discharge capacity than rectangular weirs(Akan and Iyer, 2021). Consequently, triangular weirs are not suitable for measuring large flow rates. From a hydraulic standpoint, achieving the same flow rate through a triangular weir requires a 30% increase in the upstream head compared to a rectangular weir (Akan and Iyer, 2021). To overcome these limitations, trapezoidal weirs, which combine features of both rectangular and triangular shapes, have been proposed as an effective solution. The Cipolletti weir is a notable example of this design, successfully balancing measurement accuracy and discharge capacity (Akan and Iyer, 2021). Recent advancements in hydraulics have deepened our understanding of flow dynamics and the design of hydraulic structures. Research on crown wheel spillways highlights crucial hydrodynamic processes that influence discharge coefficients. Lashkar-Ara and Sheikhi (2022) investigate the performance of spillways under free flow conditions, while Habibi and Lashkar-Ara (2018) examine how floating debris accumulation affects hydraulic efficiency, finding that increased slenderness coefficients reduce overflow blockage and enhance discharge capacity. Additionally, denil fishways are explored as effective solutions for facilitating fish migration, utilizing baffles to minimize turbulence (Mahmoudian et al., 2023). Furthermore, the analysis of shear stress distributions in rectangular open channels (Lashkar-Ara and Fathi-Moghadam, 2023) provides critical insights into the impact of geometric and physical factors on boundary shear. Collectively, these studies advance our understanding of hydraulic parameters and support the optimization of hydraulic designs and modeling techniques in the field. Another type of weir that provides a good balance of accuracy and discharge capacity is the circular (or semicircular) weir. However, limited comprehensive analytical and laboratory research on its hydraulics has been performed. The complexity of the relationship between the upstream flow head and flow rate for this type of weir poses challenges that can hinder its practical use by operators. Bos (1976) and Staus (1930) developed an equation to estimate the discharge capacity of SCCWs. They formulated the following equation (Eq. 1) using the auxiliary parameters *w* and *wi*(.

$$w = 1.467\left(\frac{h}{D}\right)^2 + 0.664\left(\frac{h}{D}\right) - 0.059 \quad (1)$$

$$w_i = \frac{4}{15}\sqrt{2g} \times w \quad (2)$$

$$Q = c_d w_i D^{2.5} \quad (3)$$

where Q is the flow rate, C_d is the discharge coefficient, h is the upstream flow head, and D is the diameter of the weir. Balachandar et al., (1991) proposed a simple method for calculating semicircular weirs' C_d. Using experimental results from previous studies, they created graphs to determine C_d across various dimensionless parameters, including the contraction ratio (D/b), the ratio of upstream flow head to the weir height (H/P), and the ratio of upstream flow head to the weir diameter (H/D). In an analytical study, Vatankhah(2010) examined the H-Q relationship of a semicircular weir (SCCW). He noted that solving the H-Q for SCCWs involves dealing with elliptic integrals, which

can be cumbersome for operators. By implementing a change of variable, he derived a simple and practical formula, demonstrating good accuracy (with a mean absolute percentage error of less than 5%). Additionally, he presented an empirical formula (Eq. 4) for these weirs' C_d.

$$C_d = \frac{0.728 + 0.240\frac{H}{P}}{1 + 0.688\sqrt{\frac{H}{P}}} \quad (4)$$

Ghadba (2011) conducted a study to investigate the hydraulic characteristics of flow in SCCWs and using dimensional analysis, he derived the parameters influencing the C_d. To validate his analytical results, Ghadba (2011) performed experiments on 16 weir models with varying crest heights and radii. His findings revealed that changes in the weir radius had a more significant impact than crest height; specifically, as the weir radius increased, the flow volume over the crest also increased. Ghobadian and Meratifashi (2012) proposed a modified theoretical relationship for the H-Q relationship of SCCWs. They presented an empirical H-Q relationship that accounts for flow pattern complexity. They conducted experiments on six semicircular weirs with varying diameters and heights, reporting a correlation coefficient of 0.889 between experimental and theoretical results. Irzooki et al. (2014) conducted a laboratory study on the flow parameters of SCCWs, utilizing weirs with four different radii and three crest heights. Their results indicated that, under consistent conditions, an increase in weir radius led to a higher C_d, while an increase in the height of the SCCW resulted in decreased C_d.

As noted, deriving the H-Q relationship for SCCWs poses challenges, which accounts for the limited research on this type of weir. This study aims to derive the H-Q relationship in a simplified manner using Simpson's rule, apply dimensional analysis through Buckingham's Π-Theory to identify the dimensionless parameters affecting C_d, and ultimately propose an empirical relationship for calculating C_d.

Material and Methods

This study aims to provide a simple H-Q relationship for semicircular weirs (SCCWs). A schematic sketch of the SCCW is shown in Fig. (1). As shown in this figure, R indicates the radius of the weir crest cross-section, the flow depth over the crest by y, and the weir height by P.

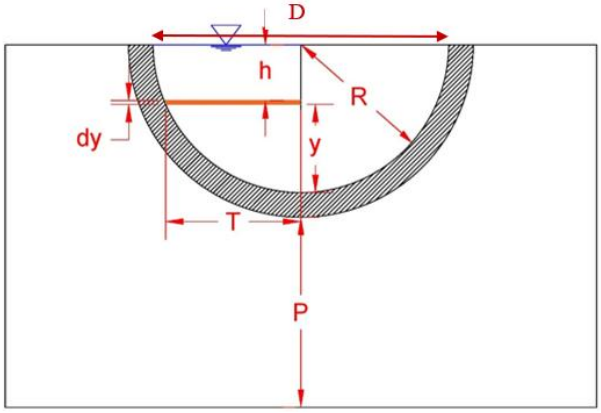


Fig. 1- Sketch of the semicircular sharp-crested weir

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To derive the H-Q relationship, referring to Figure (1), A thin element with a thickness of *dy* and width *T* is considered within the cross-sectional area of the flow passing through the weir. The theoretical discharge (Q_{th}) through this element is calculated using Equation (5).

$$\mathrm{Q_{th}} = 2\int_{\mathrm{o}}^{\mathrm{R}} u\mathrm{T}\mathrm{dy} \tag{5}$$

Where u is the velocity of the considered element flow (determined using the Bernoulli equation: Eq. 6).

$$u \cong \sqrt{2gh} \tag{6}$$

Where *h* is the upstream flow head over the element. Substituting Equation (6) into Equation (5) yields the integral form of the H-Q relationship, presented in Equation (7).

$$\begin{aligned} Q_{th} = 2\int_{o}^{R} uT\,dy \xrightarrow{u\cong\sqrt{2gh}} Q_{th} \\ = 2\int_{0}^{R} \sqrt{2gh}\ T\ dy \\ Q_{th} = 2\sqrt{2g}\int_{0}^{R} \sqrt{h}\ T\ dy \end{aligned} \tag{7}$$

As given in Equation (7), this integral ranges from zero to the radius of the crest (R), meaning the flow depth can vary from zero to *R*. To solve Equation (7), both parameters of *h* and *T* are expressed as functions of the flow depth (*y*). Since the cross-sectional geometry is semicircular, there is a direct relationship between *h*, *T*, and *y*, as indicated in Equation (8).

$$\begin{aligned} &T^2 + (R-y)^2 = R^2 \\ &T = \sqrt{2yR - y^2} \\ &h = R - y \end{aligned} \tag{8}$$

By substituting Equation (8) into Equation (7), Equation (9) is obtained.

$$Q_{th} = 2\sqrt{2g}\int_{0}^{R} \sqrt{R-y}\sqrt{2Ry-y^2}\,dy \tag{9}$$

Solving equation (9) leads to an elliptical integral, which is not easy to use. One way to simplify such an integral is to use Simpson's rule. Simpson's rule Chapra and Canale, (2011) evaluates the given integral, which requires numerical approximation. Simpson's rule is a numerical method for approximating definite integrals (Equation 10). It involves dividing the integration interval into smaller subintervals and approximating the function in each subinterval using a second-degree polynomial.

$$\begin{aligned} \int_a^b f(y)dy \approx \frac{h}{3}\Bigg(& f(y_0) \\ & + 4\sum_{i=1,3,5,\dots}^{n-1} f(y_i) \\ & + 2\sum_{i=2,4,6,\dots}^{n} f(y_i) \\ & + f(y_n)\Bigg) \end{aligned} \tag{10}$$

$$\begin{aligned} \int_a^b f(y)dy \approx \frac{h}{3}\big(& f(y_0) + 4f(y_1) \\ & + 2f(y_2) + 4f(y_3) \\ & + \cdots + 2f(y_{n-2}) \\ & + 4f(y_{n-1}) \\ & + f(y_n)\big) \end{aligned}$$

where $h = \frac{b-a}{n}$is the width of each subinterval, n is the number of subintervals (which must be even), and $y_i = a + ih, i = 0,1,2,\cdots,n$ are the discretized integration points and $f(y_i) = \sqrt{R-y_i}\sqrt{2Ry_i - y_i^2}$.

Due to the certain simplifications, such as ignoring energy loss and estimating integral parameters with Simpson's rule, the results from the theoretical equation may differ from the experimental data (measured discharge: Q_m). As a result, the values obtained from the theoretical equation (Q_{th}) are multiplied by a coefficient known as the discharge coefficient (C_d), as shown in Equation (11).

$$Q_m = C_d Q_{th} \tag{11}$$

The C_d, as given in Eq. (12), depends on the upstream flow head (h), the radius (R) of the crest, and the height of the weir (P). Additionally, C_d is influenced by the flow regime (turbulent and subcritical flow conditions, represented by flow depth y and velocity V) and flow properties (viscosity μ, density ρ, and surface tension σ).

$$C_d = \varphi(y, P, R, V, \mu, \rho, \sigma) \quad (12)$$

Using Buckingham Π theory, the dimensionless parameters affecting the C_d are derived (Eq. 13).

$$C_d = \varphi\left(\frac{H}{P}, \frac{H}{D}, \frac{D}{P}, Fr, Re, We\right) \quad (13)$$

where H represents the total head of the flow over the crest, which encompasses both the flow depth over the crest and the velocity head of the approaching flow. Therefore, instead of using y (the flow depth), H, which denotes the total head, is used.

Evaluation of the Developed Model

After developing the numerical model based on Simpson's rule, its results were evaluated using the statistical index MAPE. The MAPE (Mean Absolute Percentage Error) is the mean of the absolute percentage errors and is one of the most commonly used indicators for assessing the accuracy of predictive or estimation models. This metric is widely applied in time series forecasting, regression modeling, and numerical model analysis. The MAPE index is defined by the following equation(Neysi et al, 2025):

$$MAPE = \sum_{i=1}^{n} \frac{1}{n} \left|\frac{A_i - F_i}{A_i}\right| \times 100 \quad (14)$$

In this equation, Ai represents the actual value at time, Fi denotes the predicted value by the model at time , and n is the total number of data points.

Validation of the proposed formula

Data from various researchers, including Ghobadian and Meratifashi (2012), Irzooki et al. (2014), and Ghadba (2011), were used to calibrate and validate Equation (10) against laboratory data and to determine C_d. Table (1) lists the geometric dimensions of the weirs and the hydraulic parameters from their experiments.

Results and discussion

This section presents the calibration results of the proposed formula (Equation 10) and the discharge coefficient (C_d) calculation. The equation was calibrated using experimental data derived from over 43 laboratory models comprising 240 experimental runs. Selected calibration results for various weir geometries are shown in Figure (2), which demonstrates a strong correlation between the observed data and the results generated by the proposed formula. To calibrate the proposed formula with the laboratory data, 10 subinterval was considered. Notably the modeling accuracy was approximately 7%.

Table 1- The range of geometric and hydraulic data collected concerning SCCWs

Authors	Range	P(m)	D(m)	Q(l/s)
Ghobadian and Meratifashi (2012)	min	0.198	0.15	1.8
	max	0.25	0.25	15.7
Irzooki et al. 2014	min	0.14	0.31	1.34
	max	0.248	0.515	28.86
Ghadba (2011)	min	0.15	0.1	1.64
	max	0.3	0.4	46.38

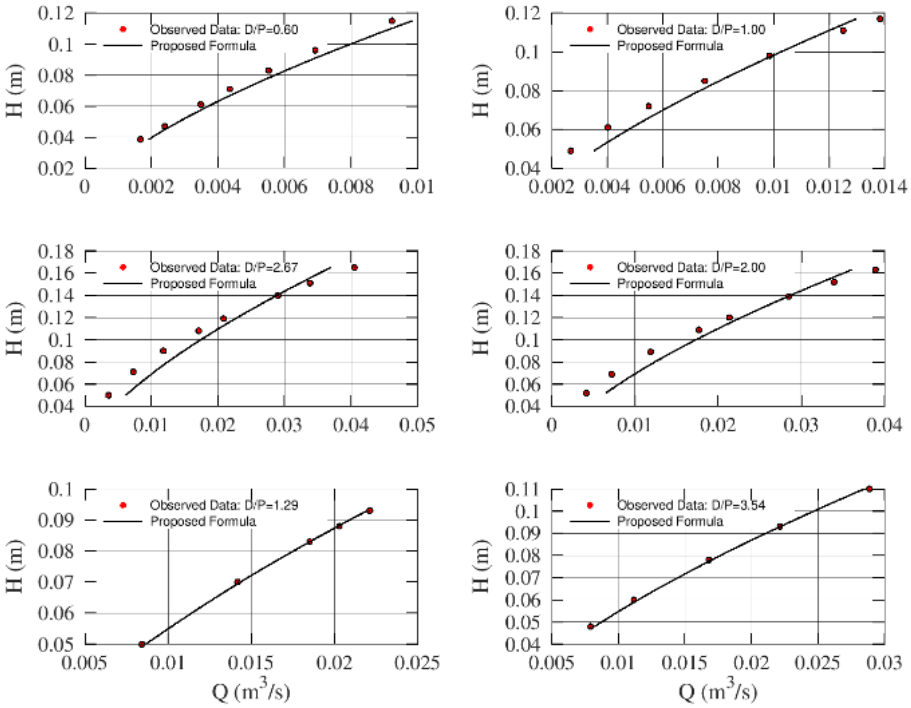


Fig. 2 – H-Q relation of SCCWs: observed data and proposed formula

During the calibration of Eq. (10) for each laboratory weir model, a value of C_d was determined for each relative upstream head, with values ranging from 0.6 to 1.10.

As shown in Figure 4, the discharge coefficient exhibits a direct logarithmic relationship with the relative flow head parameters. Initially, its value increases and then remains constant.The semicircular geometry of the weir causes the discharge coefficient to initially increase and then stabilize at an almost constant value. As the H/P ratio increases, energy losses become greater, leading to a reduction in the discharge coefficient. However, with further increases in H/P, since both energy loss and flow velocity increase simultaneously, their combined effect results in little to no change in the discharge coefficient. This behavior has also been observed in the studies by Ghobadian and Meratifashi (2012) and Irzooki et al. (2014).

The histogram and violin plot of the C_d values are presented in Figure (3).

As illustrated in Figure (3), the average value of C_d is approximately 0.96 (≈1), with a 95% confidence interval ranging from 0.95 to 0.97. As detailed in the Materials and Methods section, various parameters influence C_d (Eq. 13), including H/P, H/D, and D/P. The values of C_d for the same models (shown in Figure 2) were plotted versus H/P and are shown in Figure (4).

According to Eq. (13), in addition to H/P, the geometric parameter D/P also influences C_d. To illustrate the variation of C_d regarding D/P, an average value of C_d was calculated for each laboratory model, and the results were plotted against D/P, as shown in Fig. (5). This figure demonstrates that for nearly all models, the average value of C_d is clustered around one.

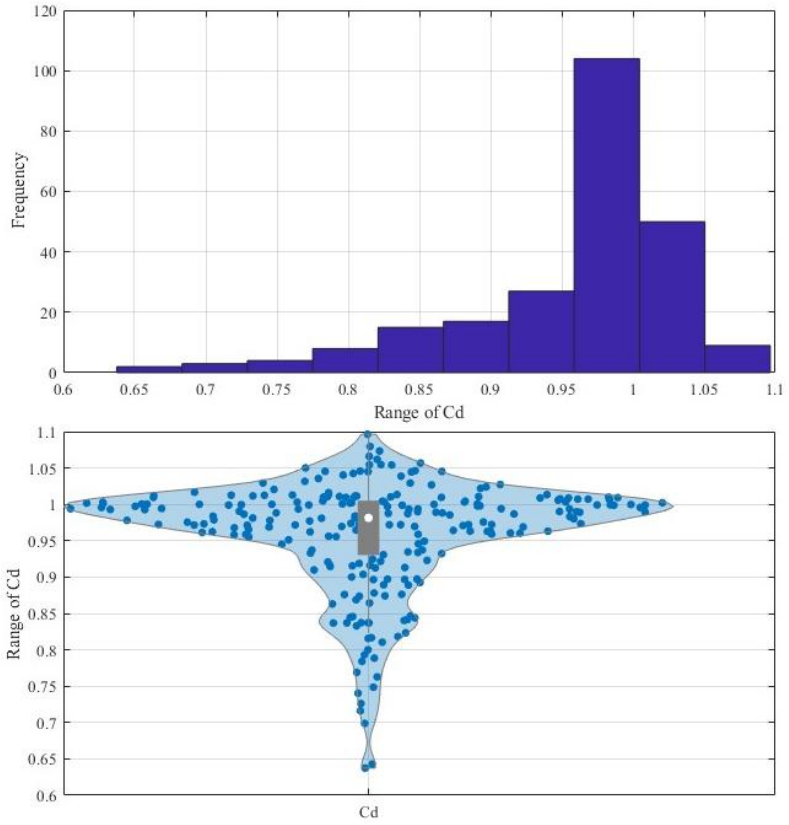


Fig. 3- The histogram and violin graph of the obtained C_d of SCCWs

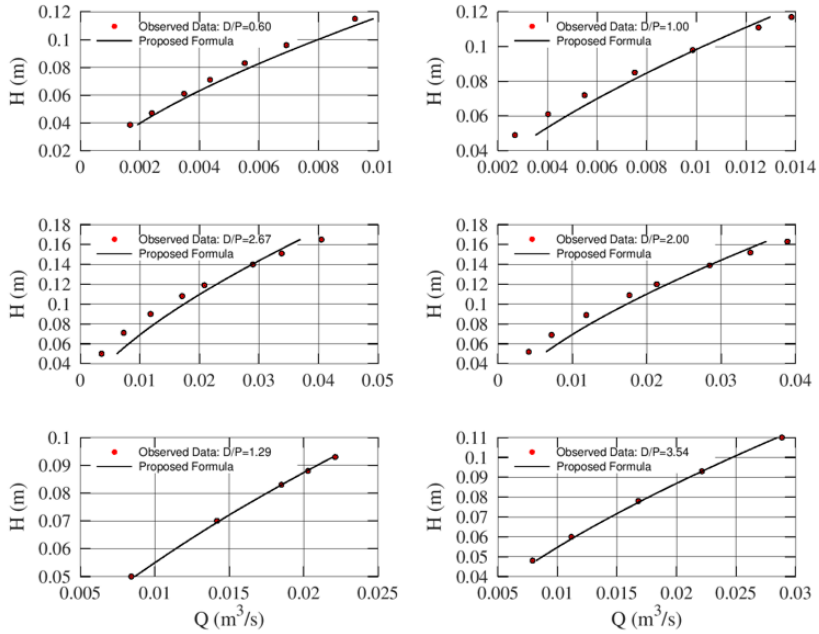


Fig. 4- The values of C_d versus the H/P for some models of SCCWs

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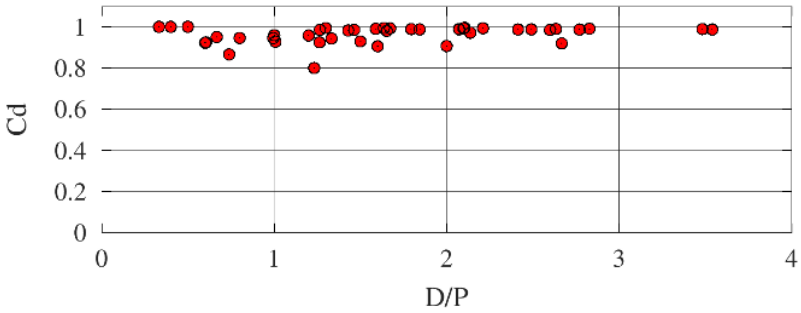


Fig. 5 – The average values of C_d of SCCWs versus the D/P

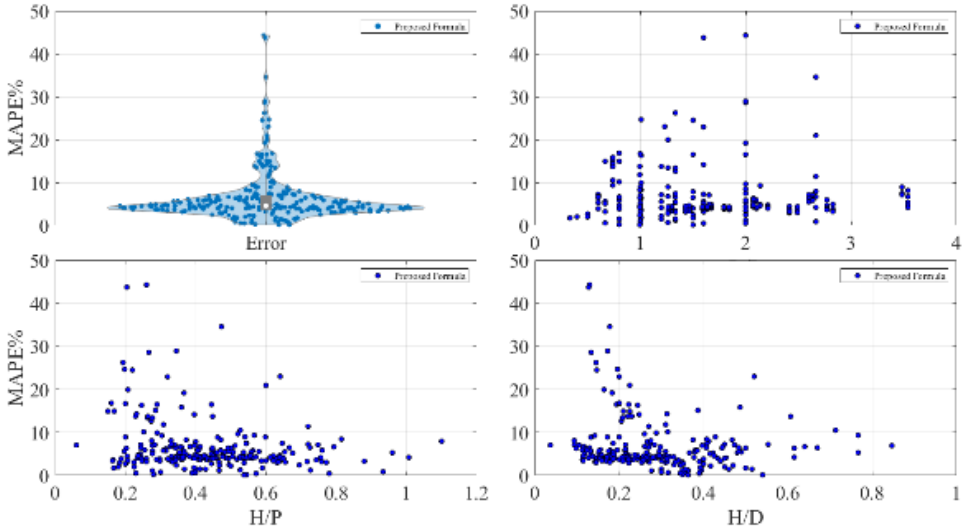


Fig. 6 – The values of the MAPE of the proposed formula versus the involved parameters

The developed formula for C_d

In this part, according to Table 1 and using the nonlinear regression technique, a nonlinear equation given in Eq. (14) was proposed to calculate the C_d for semicircular sharp-crested weirs.

$$Cd = \left(\frac{H}{P}\right)^{0.31\frac{D}{P}} \quad (14)$$

As presented in Table 1, Eq. (14) was utilized for D/P between 0.33 and 3.54, while H/P ranges between 0.06 and 1.10. Noteworthy that in the same condition (same ranges of D/P and H/P), the H/D changes between 0.34 and 0.85. The proposed formula's average error (MAPE) is about 7.4 percent (%). The Violon graph of MAPE results of the proposed formula and its variation versus the involved parameters (H/P, H/D, and D/P) are shown in Fig. (6). As shown in Fig. 6, most errors accumulated between 0 and 10 percent. Increasing the H/P or H/D reduces the MAPE (error of the proposed formula). Fig. 6 shows that no significant relation exists between the variation of H/P and MAPE.

Compared with previous research

In this section, the results of this study are compared with those of previous researchers. For this purpose, the results of two researchers, including Vatankhah(2010) and "Staus (1930)" (Bos, 1976), were selected. As mentioned, for any weir, including SCCWs, two essential components are the H-Q and the

C_d. The results of the C_d based on the two approaches of the mentioned researchers are presented in Fig. (7). As is evident from these figures, the range of the calculated C_d based on the Vatankhah(2010) and Staus (1930) approaches for the data published by Ghobadian and Meratifashi (2012) and Ghadba (2011) is in the range of 0.6 and 0.7, while the results of C_d calculated by "Staus(1930)" approach for Irzooki et al.(2014) ranges between 1.00 and 2.54.

The value of C_d calculated by the proposed formula is between 0.6 and 0.9, and its average value is about 0.96; at a 95% confidence level, it is between 0.95 and 0.97. Well, of course, the difference between the range of the proposed formula and the range of calculated C_d by the approaches of Vatankhah (2010) and Staus(1930) is due to the final equation and the powers considered for the flow head. The H-Q curve was considered for a more accurate comparison of these formulas. The performance of the proposed formulas (all tested formulas in this study) for the H-Q curve of all the models was evaluated. For example, the results of six modules (presented in Figure 2) are shown in Figure (8). On average, the MAPE of Vatankhah (2010) formula is about 3 percent, the error of the Staus(1930) approach is about 5 percent, and the error of the formula presented in this study is about seven percent.

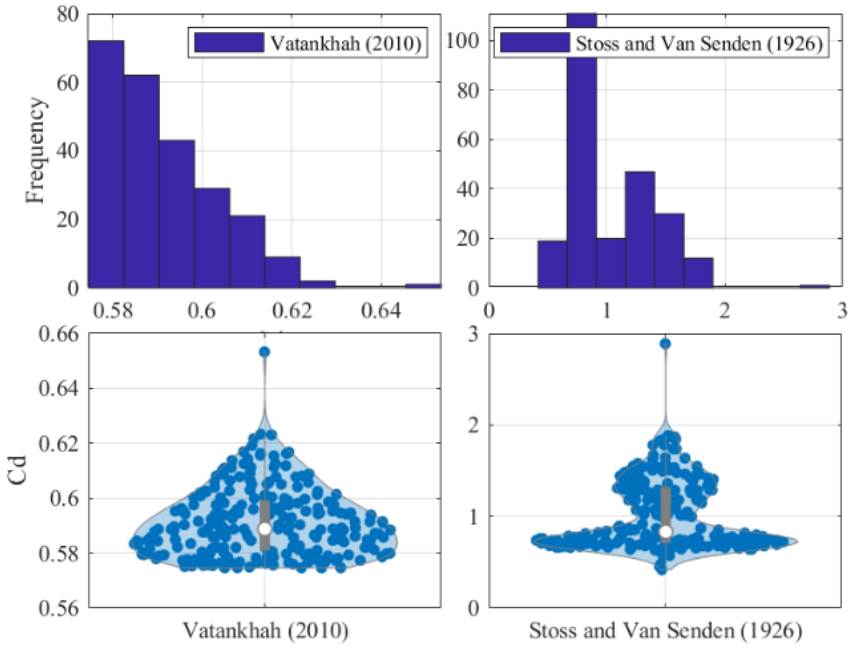


Fig. 7 – The histogram and violin graph of the obtained C_d of the semicircular sharp-crested weir

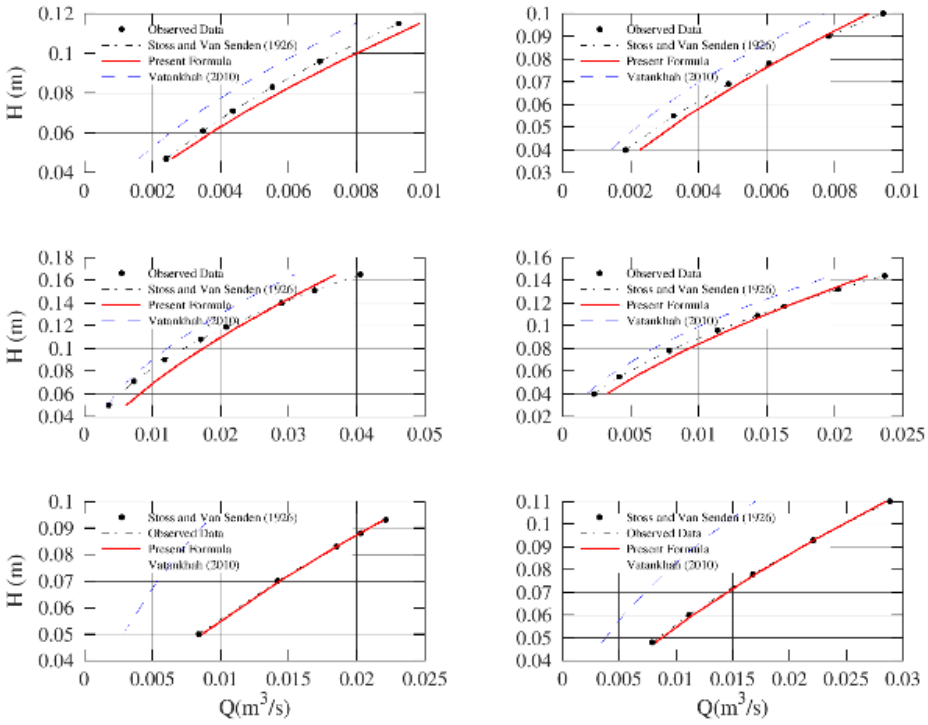


Fig. 8- The results of head discharge simulation through the present study, Vatankhah(2010), and Staus (1930)

Conclusion

Here, an effort has been made to present a straightforward solution to the integral form of the H-Q curve for SCCWs. To achieve this, both components under the square root in the integral form of the H-Q equation were approximated using Simpson's rule. The resulting formula was calibrated against various geometries of SCCWs. Notably, the variable geometric parameters for this type of weir are the crest's diameter and the weir's height. The results indicate a good agreement between the proposed formula and the observed data (average error of about 7.5%). Dimensional analysis reveals that two parameters, including the relative upstream head (H/P) and the ratio of the crest diameter to the height of the weir (D/P), are involved in C_d. The findings indicate that C_d varies between 0.6 and 1.1, with an average value of approximately 0.96, accompanied by a 95% confidence interval of (0.95, 0.97), considering the ranges of H/P between 0.06 and 1.1, and D/P between 0.33 and 3.5. Plotting the average discharge coefficient against D/P demonstrates no significant trend in their variations. For most values of D/P, C_d remains approximately 0.96.

Notations

Q_m	measured discharge
Q_{th}	theoretical discharge
C_d	discharge coefficient
D	Diameter
dy	Thickness of an element
H,h	Total upstream head
H-Q	Head discharge
MAPE	mean absolute percentage error
max	Maximum
min	Minimum
P	Weir Height
Q	discharge
R	Radius
SCCWs	Semicircular sharp-crested weirs
T	The top width of the surface of the flow

u	Flow velocity
y	Flow depth

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Conflict of Interest

The authors declared no potential conflicts of interest concerning the research, authorship, and publication of this article. Confirmation.

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Data Availability Statements

Data Availability Statement: All information and results are presented in the text of the article.

Authors' contribution

First Author: Abbas Parsaie: idea, writing.
Second Author: Mohammad Bahram Yarahmadi: edit and revise
Third Author: Mahziar Basit Nejad: collection of data, revise.

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